

Momentum First

Arne Klaveness

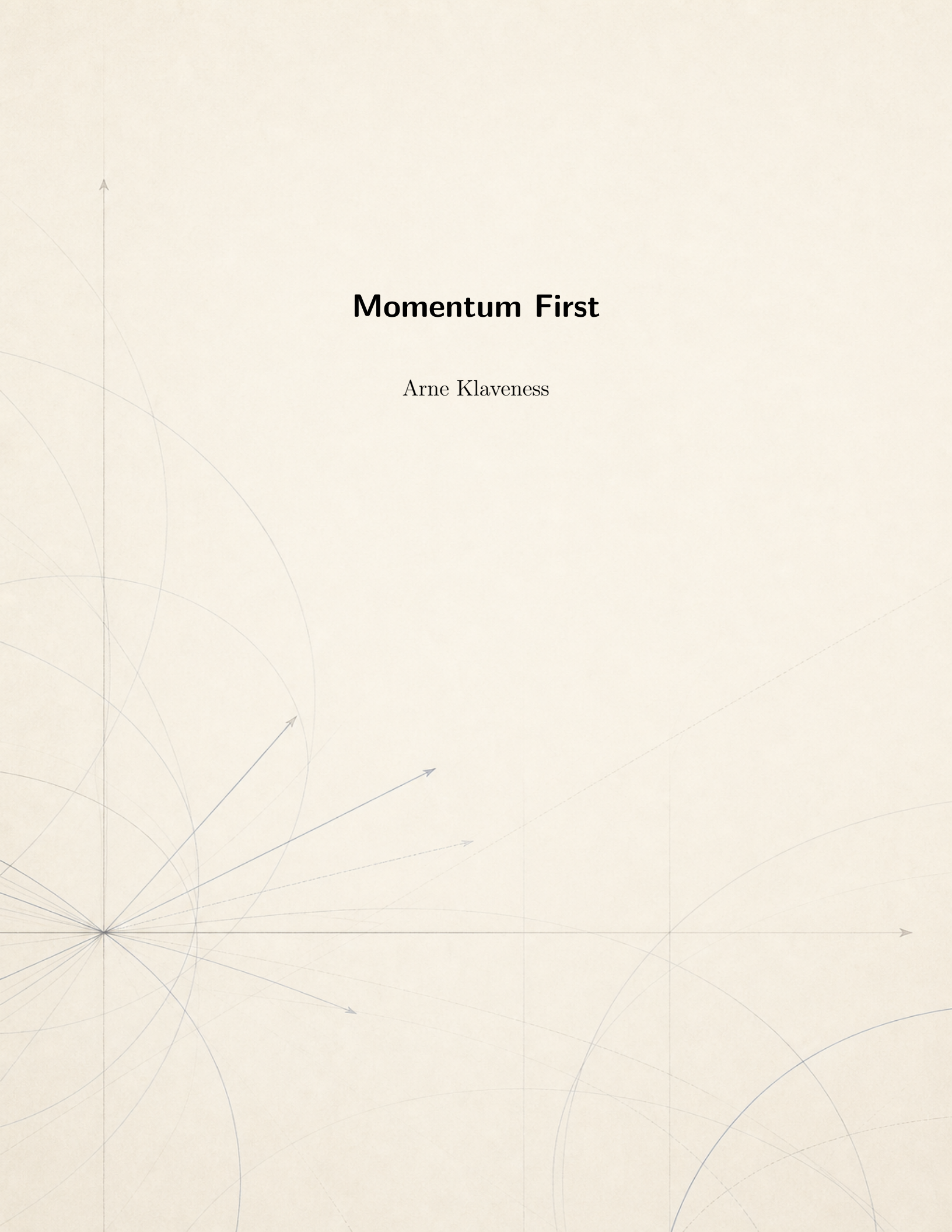


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Momentum First

Preface

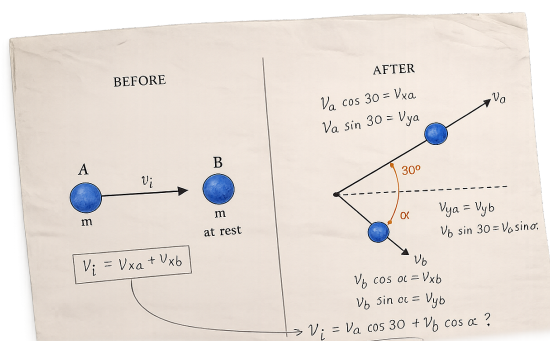
A Persistent Tension

I can trace the beginning of this project back to a small moment in middle school.

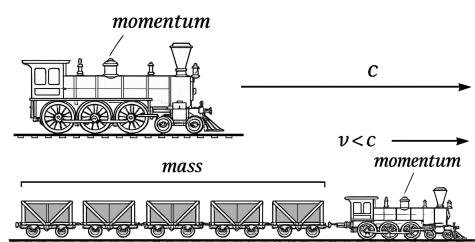
We were solving a scattering problem with two bodies, and I was stuck. I had been trying to get the result from conservation of momentum alone, but I could not make it work. Eventually I went to the teacher and asked for help. He looked over the problem, glanced at my attempt, and then explained—with that patience teachers learn to master—that I had to use both conservation of momentum and conservation of energy. That made it easy. The algebra did its duty, the equations closed, and the result landed.

But it also planted a tension in me that never really left.

Both momentum and energy were conserved? Why both? To me, they seemed to point toward the same thing. Both of them, somehow, had to do with motion. Yet here they were: two different conserved quantities, both necessary, both fundamental. I remember the feeling clearly. Not that physics had become clearer, but that something about it had become stranger. I did not know what to do with that thought at the time. I only know that I never quite got over it.



The same tension resurfaced years later at university. We were studying the elastic scattering of a photon traveling along the (z)-axis toward a stationary electron.



By then I had developed a private picture of momentum as something like the locomotive of directed motion, and mass as its wagons. It was a simple model—too simple, I suspected, to be literally true—but it helped me think about why light should travel at (c), and why a particle with mass should never exceed it. It was, at best, a rough fit with special relativity. Still, it made me linger over the details of that scattering problem.

And once again the old unease returned.

After the collision, the outgoing photon emerged at an angle. It had acquired a momentum component perpendicular to its original motion. If one changed the parameters, that perpendicular component could even dominate the outcome. The electron, in turn, acquired an equal and opposite component of its own.

Where had that perpendicular momentum come from?

That question bothered me far more than it probably should have. I found myself resisting the idea that momentum simply “turns,” as if direction were casually rewritten in flight. If a perpendicular component appeared after the collision, then perhaps it had not been created in that moment at all. Perhaps it had been there in some balanced or hidden form all along, internally locked in place. In my own mental imagery: two counter-balanced locomotives.

And once that thought appears, it is difficult not to take the next step.

Maybe mass is not something fundamentally separate from momentum. Maybe mass is momentum in another form—momentum caught in an internal structure, held in a loop rather than released into ordinary travel. The thought was crude, and I knew it was crude, but it had a stubbornness to it. It would not go away.

Still, there was an obvious problem. If this picture were to make any sense at all, then the internal momentum of a moving particle could not remain untouched. It would have to swell as the particle’s ordinary momentum increased. And if that were true, then perhaps energy was not a second, mysterious “motion quantity” after all, but a measure of the total momentum content of the particle.

That was the intuition. But intuition is cheap. What I did not have was a form.

Much later, I found myself staring at the expression for relativistic energy written in momentum form:

$$E = \sqrt{m_0^2 c^4 + p^2 c^2}.$$

And there it was.

Written instead as

$$\frac{E}{c} = \sqrt{m_0^2 c^2 + p^2},$$

it struck me differently. Not as a familiar formula, but as geometry. The square root was no longer just an algebraic convenience. It looked like structure: two momentum components combined into a magnitude. For the first time, the old feeling I had been carrying around seemed to meet a mathematical expression that could actually bear its weight.

I still did not have the answer, though. I tried a few asymmetric constructions around that form, but nothing really held. At the time I was in the middle of doctoral work, and this remained what it had long been: a private line of thought that kept returning now and then. Not something I could fully commit to. I had a thesis to finish. I presented the basic idea to a couple of professors on campus, received little more than a shrug, and left it there.

Twenty years passed before I found myself with a bit of spare time on my hands.

By then, AI had arrived, and I thought I might as well give it another try—just for fun. I quickly built a numerical scattering test repository so I could start probing candidate expressions directly. Now the question could be pushed against explicit checks rather than tedious algebraic gymnastics.

For a while, I just kept missing. Then a thought surfaced that, in retrospect, seems almost embarrassingly simple: if motion introduces an anisotropy in the momentum structure, then that anisotropy cannot stand alone. An asymmetry in the direction of travel must be balanced by a corresponding negative asymmetry in the opposite direction. Otherwise two particles colliding inelastically and coming to rest could not recover a fully symmetric momentum structure—a sphere.

That suggested that the anisotropic contributions themselves had to come in a symmetric pair.

The simplest form I found was

$$p_+ = \sqrt{m_0^2 c^2 + p^2} + \frac{p}{2}, \quad p_- = \sqrt{m_0^2 c^2 + p^2} - \frac{p}{2}.$$

And for the first time, the tests stopped fighting me.

That was not yet a finished framework. It did not suddenly answer every question. But it was the first moment in all those years when the old tension stopped feeling like a private irritation and started to look like the beginning of an actual structure.

This book grows out of that tension.

What Kind of Book This Is

This is not a conventional textbook, and it is not a finished theory presented as settled doctrine. It is an attempt to develop a framework in public and in order: to state its primitive assumptions clearly, to see what follows from them, and to keep the difference visible between what is genuinely secured and what remains partial, suggestive, or provisional.

That distinction matters here. A framework like this becomes useless if every resemblance is treated as an explanation, every interpretation as a conclusion, or every promising structure

as a result. If this book is worth anything, it will be because the reader can tell where the structure is strong, where it is provisional, and where it may simply fail.

On Authorship and Collaboration

This book is authored and released by **Arne Klaveness** (ORCID 0009-0004-1536-3055).

The project is developed in an environment where human and AI contributors may assist through draft text, criticism, review, derivation support, editing, figures, repository automation, and structural proposals. These are treated as real contributions rather than as invisible assistance. Authorship, however, is not the same thing as contribution. The manuscript keeps a stable human authorship line, while broader project contributions are acknowledged separately.

In particular, this manuscript has been developed with AI-assisted drafting, review, editing, and repository automation support. AI-assisted contribution is credited as such, not as formal authorship or DOI creatorship. All claims, interpretations, and release decisions remain the responsibility of the human author.

Contributions do not become part of the book automatically. They enter through review and verification. That verification may be carried out by human or AI reviewers, but not every review counts equally: a verifier must meet the project’s capability threshold, and the review must be appropriate to the material under consideration.

A chapter-level status ledger is maintained in the project repository and rendered in this book as the **Manuscript Status** page in the back matter. For each chapter it records how mature the material is and what level of verification it has passed, so that early or unverified chapters can be read as provisional. The ledger uses names for human contributors and the label **AI-assisted** for non-human assistance. The book itself keeps the distinction visible between authorship, broader contribution, and later verification.

How to Read This Book

There is no single path through this book, but there is a shared spine. The foundations—the primitive variables, ADMC, and the invertible mappings—carry everything downstream. Wherever your interest ends up, start there. The later material on gravity, quantum structure, and cosmology leans so heavily on those early locks that, read out of order, the downstream claims tend to look either stronger or weaker than they actually are.

Where you go **after** the foundations depends on what you came for:

- **For the idea**—the Preface, the Introduction, and the opening of the Foundations section are enough to see what question is being asked, what shift is being proposed, and why the framework is arranged as it is.
- **For the machinery**—move straight through the foundational derivations and their first structural consequences. That is where the framework earns or loses its credibility.
- **For the applications**—gravity, quantum structure, cosmology—read those sections against the locks they depend on, not on their own terms.

However you read, read **critically**. Watch for where the text establishes something directly and where it only recovers a familiar pattern: reproducing a known structure is not the same as explaining it from first principles. Hold the assumptions, scope conditions, regime claims, and falsifiers up to the light—those are the pressure points. A framework like this should not be judged by whether it sounds intriguing, but by whether it makes clear claims and can survive contact with them.

Working Method

What follows should be read as an attempt at ordered construction rather than proclamation. The question is not whether a momentum-first language can sound appealing. The question is whether it can carry weight.

That means beginning with primitive assumptions, following the structure they actually support, and resisting the temptation to claim more than the framework has earned. If the project is worth anything, it will be because the reader can see where the structure is strong, where it is incomplete, and where it may fail.

Part I

Tier 1 — Foundations

1 Introduction

1.1 The Success of Physics

Any attempt to rethink foundations should begin with respect.

Modern physics is not a damaged structure waiting for rescue. It is one of the great intellectual achievements: mathematically precise, experimentally powerful, and remarkably successful at organizing the behavior of the physical world. Classical mechanics, relativity, quantum theory, and field theory have earned their authority.

That success sets the standard here. An alternative starting point does not earn attention by sounding cleaner, simpler, or more philosophically appealing. It earns attention only if it can carry real explanatory weight while remaining answerable to the successes already achieved.

But success does not settle every foundational question. A theory may work magnificently while still leaving open which of its concepts are primitive, which are derived, and which sit at the center mainly because of the historical order in which the theory was built. Practical indispensability and foundational priority are not always the same thing.

This book begins in that gap. It does not start from the claim that standard physics has failed. It starts from the possibility that some familiar structures may stand too high in the explanatory hierarchy, and that beginning elsewhere may reveal a cleaner organization, and perhaps consequences that are otherwise harder to see.

1.2 Open Questions and Conceptual Pressure

That possibility becomes harder to ignore once one asks not only how the standard framework works, but what role its central concepts are actually playing.

Consider energy. Few quantities in physics are more useful. It organizes dynamics, anchors conservation laws, and travels easily across nearly every domain of the subject. Yet the closer one moves to the question of what energy is, rather than how it is calculated or conserved, the less transparent its structural role becomes. We know how to use it with great effectiveness. It is less obvious why this particular quantity should occupy so central a place in the deepest organization of the theory.

Time produces a similar pressure. Physics measures time superbly and uses it with extraordinary sophistication. But the question of what time is, as distinct from how clocks behave or how equations are parameterized, still pushes the theory toward competing interpretations. Is time fundamental, emergent, relational, observer-bound, or something else again? Those are not decorative philosophical questions. They reach into the way the theory is organized.

The issue, then, is not a lack of mathematics. It is almost the opposite. Physics is so formally successful that the difference between primitive structure and successful packaging can become hard to see. A concept may be indispensable in practice while remaining less clear than the role it is asked to play.

Once that tension is visible, reconsidering the order of explanation becomes a serious question rather than a stylistic preference. That is the pressure point from which M1 begins.

1.3 The Momentum-First Wager

M1 begins from a simple but consequential question. If some of the deepest conceptual pressure in physics comes from the place assigned to its central quantities, what happens if one starts from momentum instead? Not momentum as one useful conserved quantity among others, but directional momentum conservation as the primary structural demand.

In the formal development of the framework, that move takes the form of **Additive Directional Momentum Conservation** (ADMC). The claim is that if directional momentum conservation is required first, a specific momentum structure for particles is forced within the regular additive and symmetry-compatible class developed in Foundations, and what standard physics treats as separate conservation laws for energy and momentum can be read as expressions of one underlying conservation structure. On this view, energy is not a second independent conserved primitive, but a measure of total momentum content. Foundations §2.3 and Derivation 2.3A make that scoped result precise.

The same reordering changes the status of **time**. In M1, time is not introduced as an independent primitive dimension. It is read instead as a measure of the observed rate of change. That proposal matters only if the framework can earn it, not as a verbal reinterpretation, but as part of a structure that becomes clearer when the momentum-first move is taken seriously.

That is the wager. If the move is sound, the primitive set should tighten, the relation among mass, energy, and momentum should become clearer, and structures that often appear separately should begin to fall into a more intelligible order. If it does not deliver those gains, M1 has not earned its departure.

The book therefore treats M1 neither as a slogan nor as a finished doctrine. It treats it as a disciplined attempt to put directional momentum conservation first and then follow the consequences wherever that choice genuinely leads.

1.4 What This Book Is Trying to Do

If the momentum-first wager is to be judged fairly, it must be developed in an order that makes visible what is assumed, what is built, and what the framework can actually carry. This book is not organized to protect the proposal from difficulty. It is organized to expose its strength, its limits, and, where necessary, its failure.

It therefore begins with the core structure. The first task is to make the primitive terms, conservation principles, and formal mappings precise enough that the framework can stand, if it can stand at all, on its own terms. Breadth comes later. Foundations come first because without them the rest would be rhetoric.

Only after that does the argument move outward toward relativity, gravity, quantum structure, and cosmology. And as it moves outward, the book does not assume in advance that every continuation will be equally secure. Some lines of development may hold firmly. Some may work only in a limited regime. Some may force a revision, or even a genuine fork, when two incompatible continuations both deserve to be seen clearly before one is chosen.

The book should therefore be read neither as a finished doctrine nor as a sequence of rhetorical claims. It is a structured investigation. Its successes are not all of one kind, and it matters to distinguish what is directly built from what is only recovered, what is interpretive, and what remains exploratory.

The standard, however, is not loose. M1 earns its place only if this momentum-first order does real work: if it reduces the genuine primitive burden, clarifies the relation among mass, energy, and momentum, recovers known physics where it claims recovery, and reveals structure that is otherwise harder to see. It fails if the simplification is cosmetic, if the framework quietly assumes what it claims to explain, or if the correspondence burden overwhelms the primitive proposal.

2 Foundations

2.1 Core Momentum Terms and Directional Notation

This section introduces M1's three core momentum terms and fixes the directional notation built from them. It serves as a compact reference for what follows; the next section develops the physical configuration these symbols describe.

2.1.1 Fermic momentum

Fermic momentum, written p_f , is the nonnegative intrinsic momentum scale associated with stable internal cycling. At the inertial baseline,

$$p_f = m_0 c,$$

where m_0 labels the particle's rest-state mass and c is the invariant local propagation scale. M1 reads the corresponding mass content as momentum locked into the internal cycle associated with the particle's identity.

2.1.2 Botic momentum

Botic momentum, written p , is the nonnegative momentum quantity through which the particle's internal cycle carries translational expression. Its organization includes both a spherical contribution and an asymmetry with a net direction.

The quantity p remains the botic momentum throughout M1. Its vector and component forms describe its directional asymmetry; they do not replace it as the foundational quantity.

2.1.3 Core momentum

Core momentum, written M , is the nonnegative scalar measure of the particle's total momentum content. In the inertial baseline,

$$M = \sqrt{p_f^2 + p^2}.$$

This is the inertial composition rule for fermic and bosic momentum. Its physical meaning is developed in the next section.

2.1.4 Direction and projection

The net direction of the bosic asymmetry is represented by the vector $\vec{p}$, whose magnitude is

$$p = |\vec{p}|.$$

For an arbitrarily chosen oriented unit direction $\hat{k}$, the signed component along that direction is

$$p_k = \vec{p} \cdot \hat{k}.$$

2.1.5 Directional shell readings

Along the net asymmetry axis, the two opposed shell readings are

$$p^\pm = M \pm \frac{1}{2}p.$$

For the chosen k -axis, the corresponding readings are

$$p_k^\pm = M \pm \frac{1}{2}p_k.$$

A direction perpendicular to the net asymmetry has the reading

$$p^\perp = M.$$

Superscripts $+$ and $-$ label opposed positive readings, not arithmetic signs. The quantities p_f , p , M , $p^\pm$, $p_k^\pm$, and $p^\perp$ are nonnegative scalars; $\vec{p}$ is a vector, and p_k is its signed component along $\hat{k}$. When several particles appear, a particle label is appended as a second subscript: M_i ,

$p_{k,i}$, and $p_{\vec{k},i}^{\pm}$ denote the quantities belonging to particle i . The label is omitted when only one particle is under discussion.

2.1.6 Terminology note

The terms **fermic** and **bosic** name momentum roles in M1. **Fermionic** and **bosonic** are reserved for structure, geometry, winding, chirality, or state labels. No one-to-one identification between these momentum roles and a later particle classification is assumed here.

2.2 The Momentum Configuration

A particle can translate only because it already has an organized internal momentum structure. M1 begins with that structure and distinguishes two perpendicular geometrical aspects within it: fermionic structure, which carries the particle's intrinsic organization, and bosonic geometry, on which translational momentum is expressed. Fermic and bosic momentum name the corresponding momentum roles.

2.2.1 Fermionic structure and fermic momentum

A stable particle contains a closed fermionic momentum cycle. This is the persistent structure through which the particle retains its identity while its state of motion changes. Fermic momentum, written p_f , is the momentum locked into that cycle. It supplies the particle's intrinsic symmetric momentum scale,

$$p_f = m_0 c.$$

The relation ties fermic momentum directly to rest mass: m_0 is the mass reading of momentum confined in the fermionic cycle. Other intrinsic properties, including charge, belong to the organization of that structure rather than to translational motion. Charge is not determined by the magnitude of p_f ; it characterizes how the fermionic structure is organized. The detailed geometrical account of those identity properties lies beyond the present foundation.

With no translation, the fermic cycle has no preferred external direction. Its momentum organization is symmetric, and the particle's core momentum is entirely fermic.

2.2.2 Perpendicular bosonic geometry

In flat space, the fermionic structure supports a bosonic geometry perpendicular to the fermionic cycle. Botic momentum p resides on this geometry. The word *perpendicular* describes the internal geometrical relation between the two momentum roles; it does not place p_f along one ordinary spatial axis and p along another.

The fermionic and bosonic aspects are therefore not separate particles or detachable layers. They are perpendicular aspects of one internal particle cycle. The fermionic aspect carries the locked intrinsic scale. The bosonic aspect provides the geometry on which momentum can acquire translational expression. Together these aspects form the spherical internal particle cycle referred to throughout this chapter.

The whole configuration is measured by its core momentum M . In the flat inertial baseline, M1 takes the total core momentum to follow the composition rule

$$M = \sqrt{p_f^2 + p^2}.$$

The perpendicular geometry gives this rule its **momentum-triangle** interpretation. Fermic momentum and botic momentum form perpendicular contributions to one momentum configuration and together determine its total core momentum.

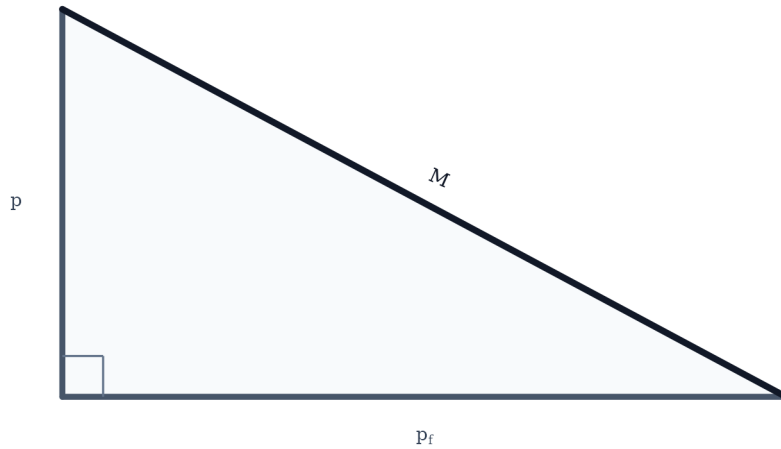


Figure 2.1: In the flat inertial baseline, botic momentum p resides on the bosonic geometry perpendicular to the fermionic cycle carrying p_f . Together they define the total core momentum M of one particle configuration.

2.2.3 Swelling, asymmetry, and translation

When a particle gains basic momentum, the whole momentum configuration swells from the fermic scale p_f to the core scale M . The swelling is the spherical contribution of p to the combined cycle. It does not yet select a direction: a larger symmetric shell alone would contain no information about which way the particle translates.

Translation appears because the bosonic structure also carries an asymmetry. Its organization over the spherical cycle is no longer directionally balanced. The shell represents that asymmetry by extending one side relative to M while contracting the opposite side, producing a net direction. Basic momentum yields translation through this asymmetry. It is therefore neither an external straight-line component nor a quantity detached from the particle's internal cycle: its spherical contribution swells the configuration, and its directional imbalance gives that momentum translational expression.

The swelling and asymmetry belong to one changed momentum configuration. They are not successive physical mechanisms. Figure 2.2 separates them only so that each can be seen. Panel (a) compares the symmetric fermic scale p_f with the swollen core scale M . Panel (b) keeps the spherical core shell M visible as a reference while encoding the basic asymmetry over the same structure. Along the net asymmetry direction, one side is displaced outward from M by $p/2$, while the opposite side is displaced inward by $p/2$, giving the complete readings $M + p/2$ and $M - p/2$. The deformation represents their organization, not a measured spatial distribution of momentum density. The local arrows describe the deformation across the spherical cycle; they do not represent momentum leaving the particle or flowing along external straight lines.

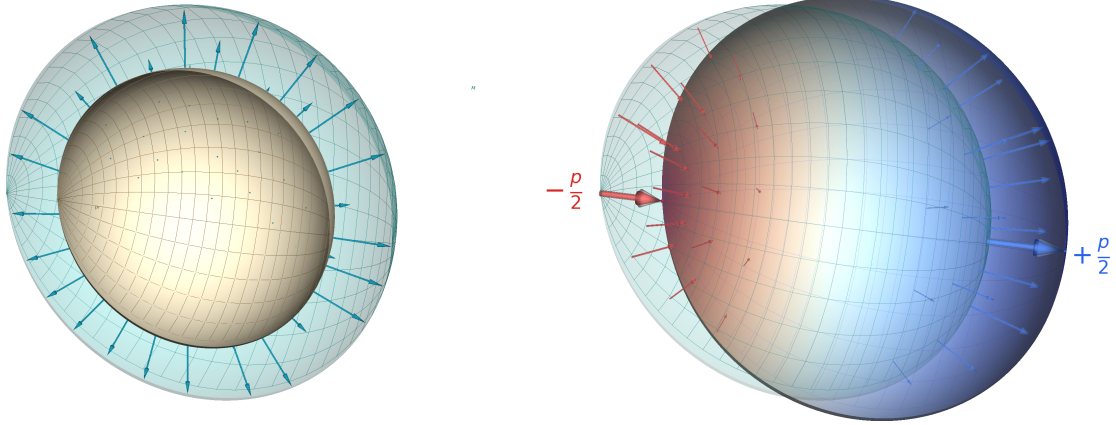
The limiting configurations make the roles clear. When no momentum is expressed as translation,

$$p = 0, \quad M = p_f.$$

The core momentum is then entirely fermic. At the purely bosic boundary of the inertial composition rule,

$$p_f = 0, \quad M = p.$$

The full structural realization of this boundary lies beyond the particle construction developed here. When both roles are present, $p_f > 0$ and $p > 0$, and the core momentum measures the complete configuration rather than either contribution alone.



- (a) The rest-state fermic scale p_f and the swollen spherical core scale M . The radial arrows compare the two shell descriptions; they do not depict a separate temporal step.
- (b) The spherical core shell M remains as the reference geometry while basic momentum p carries a directional asymmetry over the same structure. Along the net asymmetry direction, the two extremes are displaced from M by $+p/2$ and $-p/2$.

Figure 2.2: Two three-dimensional views of one internal momentum configuration. Panel (a) shows the isotropic swelling from the fermic scale p_f to the core scale M . Panel (b) shows the directional deformation carried by basic momentum p on that swollen spherical cycle. The panels separate two simultaneous features for explanation; they do not describe successive physical stages.

2.2.4 Choosing a direction

Core momentum M belongs to the configuration as a whole and does not depend on which spatial direction is chosen. The basic asymmetry has a net direction represented by the vector $\vec{p}$, with magnitude

$$p = |\vec{p}|.$$

For any chosen oriented unit direction $\hat{k}$, the signed component along that direction is

$$p_k = \vec{p} \cdot \hat{k} = p \cos \alpha,$$

where α is the angle from $\hat{k}$ to $\vec{p}$. The component p_k records the asymmetry relative to $\hat{k}$; it does not mean that basic momentum exists only along a line through the particle. Reversing the orientation from $\hat{k}$ to $-\hat{k}$ leaves M unchanged and reverses the sign of p_k .

Along the net asymmetry axis, the two opposed shell readings are

$$p^\pm = M \pm \frac{1}{2}p.$$

For the chosen k -axis, the corresponding readings are

$$p_k^\pm = M \pm \frac{1}{2}p_k.$$

Figure 2.3 connects the ordinary vector projection to these readings on the directional shell.

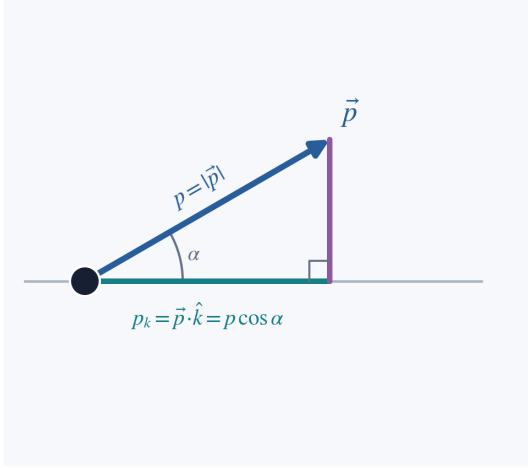
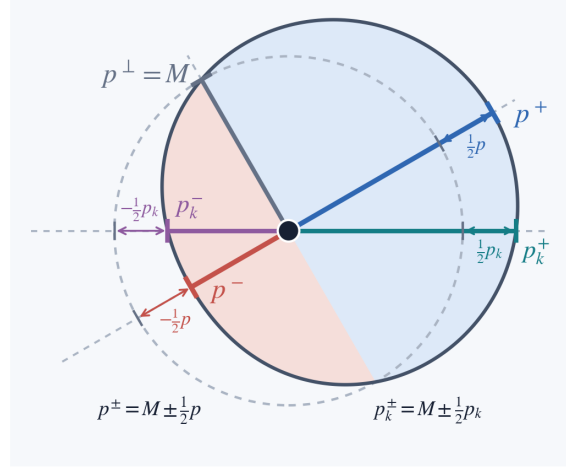
(a) Vector decomposition**(b) Shell readings**

Figure 2.3: Vector decomposition and directional shell readings. Panel (a) separates basic momentum magnitude $p = |\vec{p}|$, its represented vector $\vec{p}$, and the signed projection $p_k = \vec{p} \cdot \hat{k} = p \cos \alpha$. Panel (b) shows the net-axis readings $p^\pm = M \pm p/2$, the perpendicular reading $p^\perp = M$, and the chosen-axis readings $p_k^\pm = M \pm p_k/2$ on one directional shell. The dashed circle marks the undeformed core scale M ; capped spokes are positive scalar readings, not additional momentum vectors.

Superscripts $+$ and $-$ label the two opposed shell readings; they are not arithmetic signs. The quantities $p^\pm$, $p_k^\pm$, p_f , p , and M all denote nonnegative scalar momentum quantities. By contrast, $\vec{p}$ denotes the momentum vector and p_k its signed component along $\hat{k}$.

$$p_k^+ = p_k^- = p^\perp = M.$$

Replacing $\hat{k}$ by $-\hat{k}$ changes p_k to $-p_k$ and exchanges the two readings. The pair (p_k^+, p_k^-) therefore preserves both kinds of directional information: its common center records the positive core momentum, while its asymmetry records the signed component.

Positivity follows directly from the inertial composition rule. Since

$$|p_k| \leq p \leq M,$$

the smaller of the two readings satisfies

$$\min(p_k^+, p_k^-) = M - \frac{1}{2}|p_k| \geq \frac{1}{2}M > 0$$

for every nonempty particle configuration.

The momentum configuration has now supplied the positive directional readings that M1 conserves. The next section states that conservation principle.

2.3 Additive Directional Momentum Conservation (ADMC)

The momentum configuration developed above supplies two opposed positive shell readings, p_k^+ and p_k^- , for every oriented unit direction $\hat{k}$. M1 places conservation of momentum at the core through the following definition:

For any isolated system and any chosen direction $\hat{k}$, momentum is conserved directionally as a sum of positive quantities.

We call this principle **Additive Directional Momentum Conservation**, or **ADMC** for short. **Directional** means that the conservation statement is made relative to an arbitrarily chosen direction. **Additive** means that the conserved quantity contains one positive contribution from every particle in the system.

The conservation rule can be written as:

$$\sum_i p_{k,i}^+ = \text{constant}$$

for any direction $\hat{k}$. Using the algebraic form established in the preceding section, the same statement is

$$\sum_i (M_i + \frac{1}{2}p_{k,i}) = \text{constant}.$$

The first expression keeps the positive shell reading explicit. The second exposes the core momentum and signed directional component contained within it. ADMC applies equally to the reversed orientation $-\hat{k}$, for which each contribution is $p_k^- = M - p_k/2$.

Within the regular additive and symmetry-compatible axis-wise class, Derivation 2.3A shows that canonical normalization uniquely fixes this realized form.

The next section compares these opposite-orientation conservation statements. Their invertible relation recovers total M and signed p_k , and three independent directional components establish correspondence with special-relativistic four-momentum.

2.4 ADMC Mapping and Inertial Correspondence

The directional shell readings introduced above preserve the complete inertial information carried by M and signed p_k . The relation is linear and exactly invertible. Applied along three independent directions, it connects the M1 formulation directly to special-relativistic four-momentum.

2.4.1 Opposite-orientation map

For any oriented unit direction $\hat{k}$,

$$p_k^+ = M + \frac{1}{2}p_k.$$

Reversing the orientation changes p_k to $-p_k$ while leaving M unchanged:

$$p_k^- = M - \frac{1}{2}p_k.$$

The two readings form the linear map

$$\begin{pmatrix} p_k^+ \\ p_k^- \end{pmatrix} = \begin{pmatrix} 1 & \frac{1}{2} \\ 1 & -\frac{1}{2} \end{pmatrix} \begin{pmatrix} M \\ p_k \end{pmatrix}.$$

Its inverse is

$$\begin{pmatrix} M \\ p_k \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ 1 & -1 \end{pmatrix} \begin{pmatrix} p_k^+ \\ p_k^- \end{pmatrix},$$

or equivalently,

$$M = \frac{1}{2}(p_k^+ + p_k^-), \quad p_k = p_k^+ - p_k^-.$$

The average recovers the positive core momentum. The difference recovers the signed component. The opposite-orientation readings and the pair (M, p_k) are therefore two exactly equivalent representations of the same directional kinematic content.

2.4.2 Conservation consequences

For an isolated system, define the two system readings

$$\mathcal{P}_k^\pm = \sum_i p_{k,i}^\pm.$$

ADMC conserves $\mathcal{P}_k^+$ for every oriented unit direction $\hat{k}$ and therefore conserves $\mathcal{P}_k^-$ for the reversed orientation $-\hat{k}$. Applying the inverse map to the two system readings gives

$$\sum_i M_i = \frac{1}{2} (\mathcal{P}_k^+ + \mathcal{P}_k^-),$$

and

$$\sum_i p_{k,i} = \mathcal{P}_k^+ - \mathcal{P}_k^-.$$

Thus ADMC implies conservation of total M and total signed momentum along k . Applied along three independent spatial directions, it recovers conservation of total three-momentum together with total M . Conversely, conservation of total M and the signed components preserves the positive shell readings. At the inertial level, the two conservation formulations are equivalent.

2.4.3 Four-component form

Choose a Cartesian basis with oriented unit directions $\hat{x}$, $\hat{y}$, and $\hat{z}$. The M1 quantities can be assembled into the four-component package

$$P_+ = \begin{pmatrix} M \\ p_x^+ \\ p_y^+ \\ p_z^+ \end{pmatrix}.$$

The corresponding signed package is

$$P_M = \begin{pmatrix} M \\ p_x \\ p_y \\ p_z \end{pmatrix}.$$

They are related by the invertible transformation

$$P_+ = TP_M, \quad T = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & \frac{1}{2} & 0 & 0 \\ 1 & 0 & \frac{1}{2} & 0 \\ 1 & 0 & 0 & \frac{1}{2} \end{pmatrix},$$

with

$$P_M = T^{-1}P_+, \quad T^{-1} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -2 & 2 & 0 & 0 \\ -2 & 0 & 2 & 0 \\ -2 & 0 & 0 & 2 \end{pmatrix}.$$

In component form,

$$p_x = 2(p_x^+ - M), \quad p_y = 2(p_y^+ - M), \quad p_z = 2(p_z^+ - M).$$

The four entries of P_+ are a coordinate package for one momentum configuration, not four contributions to a single directional ADMC sum. Exact invertibility means that P_+ and P_M contain the same four degrees of inertial kinematic information.

2.4.4 Correspondence with special relativity

Standard relativistic energy is introduced through

$$E = cM.$$

The signed package then becomes

$$P_M = \begin{pmatrix} E/c \\ p_x \\ p_y \\ p_z \end{pmatrix} = P^\mu,$$

which is the standard special-relativistic four-momentum in a Cartesian inertial frame. The full correspondence is therefore

$$P_+ = TP^\mu, \quad P^\mu = T^{-1}P_+.$$

The M1 positive-reading package and special-relativistic four-momentum are related by an exact change of representation. The transformation does not discard a sign or introduce an additional degree of freedom; it reorganizes the same inertial momentum content.

The invariant relation agrees as well. M1's inertial composition rule gives

$$M^2 = p_f^2 + p^2,$$

and $p_f = m_0 c$. With metric signature $(+, -, -, -)$,

$$P^\mu P_\mu = M^2 - p^2 = p_f^2 = m_0^2 c^2.$$

Equivalently,

$$E^2 = m_0^2 c^4 + p^2 c^2,$$

the familiar special-relativistic energy-momentum relation.

The correspondence is exact at the inertial level. Opposed positive shell readings recover M and each signed component p_k ; three independent components assemble into four-momentum; and the M1 composition rule reproduces the special-relativistic invariant. M1 differs in its foundational ordering: it begins with positive momentum contributions in oriented directions and reaches the familiar four-component formulation through the invertible map.

2.5 Space and Momentum: Stage and Actor

Before turning to clocks and dilation, we need one further stance on space and momentum.

2.5.1 Space as the stage

In M1, space is the stage on which physical systems exist, persist, and change. This does not make it empty scenery. A stage has structure: it determines which relations are available, which configurations can be stable, and what counts as possible motion for a physical system.

If the stage is different, the same momentum content need not support the same trajectories, equilibria, or local kinematic behavior. In the inertial baseline, this stage is treated as given. Later chapters will ask how momentum changes the local stage. Here the needed point is simpler: space conditions the available kinematic relations.

2.5.2 Momentum as the actor

Momentum is the actor of change. Translational motion is momentum expression. Mass is read through locked fermic momentum. Interaction is described through transfer, redistribution, or reorganization of momentum content.

This is the force of the momentum-first claim. What changes in a physical system changes through momentum expressed within spatial relations, not because time acts as an independent driver.

Momentum does not have to be carried by a particle. A particle is one stable organization of momentum, but momentum can also be held in the physical structure of space itself. Space remains the stage, but it need not be passive: its structured state can, in certain situations, carry momentum and participate in its transfer.

2.5.3 Space-mediated momentum coupling - SMC

The stage and the actor are not separate worlds. Momentum is always expressed in space, and space matters because it conditions that expression.

Physical systems also do not exchange momentum across nothing. In M1, every force is understood as a mode of **space-mediated momentum coupling**: momentum content in one system becomes dynamically relevant to momentum content elsewhere through spatial relation.

This principle is deliberately general. It does not say that electromagnetism, gravity, the strong interaction, and the weak interaction are mechanically identical. They are not. It says that beneath their different mechanisms is a shared form: interaction means that momentum is communicated, constrained, redistributed, or reorganized through space.

The detailed mechanisms differ. For now, the foundation is only this: forces are not external additions to momentum-first physics. They are the ways momentum becomes coupled through space.

2.5.4 Toward clocks and dilation

This distinction now gives us the right starting point for time.

A clock is not driven by time as an independent medium. A clock is a physical system whose momentum content undergoes regular, repeatable change. Its rate depends on how that momentum is organized, coupled, and expressed within the spatial conditions available to it.

If clock rates differ, the first place to look is therefore not a changing time-substance, but a difference in the physical conditions under which the clock's momentum content can complete its cycle. The next section turns to time and dilation from that standpoint.

2.6 Time, Clocks, and the Interpretation of Dilation

The stage-and-actor distinction changes how M1 speaks about time. Space supplies the relational setting. Momentum carries the changing physical content. Time is not a third actor alongside them. It is the measure abstracted from physical change.

This reverses the usual explanatory order. M1 does not begin with time as an arena and then ask how matter moves through it. It begins with physical systems whose momentum content is organized and expressed in structured ways, then asks how clocks, durations, and temporal order are read from those changes.

2.6.1 Clocks as physical cycles

A clock is a physical system with a stable repeatable cycle. Its rate is therefore not explained by time itself, but by the structure that lets the system cycle.

In M1, the clean primary clock-carrying sector is fermic. Fermic momentum p_f is the locked intrinsic scale associated with stable internal cycling, and composite material clocks inherit their timing behavior through stable internal structure built from such cycle-carrying constituents.

Bosic momentum matters differently. It is not the intrinsic clock scale, but directed bosic expression changes how the total momentum content of a system is organized. A moving clock therefore does not merely carry the same internal cycle through an external time variable. Its fermic cycle changes relative to the free-current baseline of the surrounding space as the relation between M and p_f changes.

This is the section's central distinction: clock language belongs first to systems with stable internal cycles. Pure massless or bosic transport can still exhibit redshift, phase, and arrival-rate effects, but those should not be casually identified with internal clock slowing in the same sense.

The general cycle-duration factor is

$$\boxed{\frac{M}{p_f}}.$$

It tells how much the duration of a fermic cycle changes from that free-current baseline. The inverse ratio p_f/M is the clock-rate factor: as M/p_f increases, each cycle takes longer and the clock runs more slowly.

2.6.2 Inertial time dilation

Inertial time dilation is the simplest place where the distinction becomes quantitative. A moving clock runs slow not because time itself stretches, but because more of the system's total momentum content is expressed bosically. The ratio M/p_f increases, so the fermic cycle takes longer.

The familiar relativistic clock relation is

$$\Delta\tau = \Delta t \sqrt{1 - \frac{v^2}{c^2}}.$$

In M1 variables the same inertial relation is written

$$d\tau = dt \frac{p_f}{M}.$$

Since the inertial baseline gives

$$M^2 = p_f^2 + p^2,$$

and since $p_f = m_0 c$ while M corresponds to $E/c = \gamma m_0 c$, the ratio p_f/M is the usual inverse gamma factor,

$$\frac{p_f}{M} = \frac{1}{\gamma} = \sqrt{1 - \frac{v^2}{c^2}}.$$

The formal result is the standard inertial one. What changes is the reading: the slower clock is a slower physical cycle, caused by the system's changed momentum organization.

The ratio M/p_f is not specific to inertial motion. It governs the fermic cycle whenever the relation between bosic and fermic momentum changes. What changes from one setting to another is the physical reason: motion reorganizes the particle's momentum, gravity changes how that momentum configuration is expressed under the local spatial geometry, and expansion changes its expression as the stage evolves. Each later engine must derive how its particular structure changes M/p_f ; the cycle rate then follows from the inverse ratio p_f/M .

2.6.3 Motion and material contraction

Directed motion also changes material equilibrium structure. A rod moving along one direction is not merely observed differently, and space itself is not simply compressed around it. In the M1 reading, directed bosic expression changes the effective conditions under which the system's internal structure remains stable.

That is the constructive reading of length contraction. The same underlying physical laws are not replaced by new laws for moving matter. Rather, the moving system carries a different directional momentum organization, and its stable configuration is determined under those altered kinematic conditions. A moving rod is shorter along the direction of motion because its equilibrium structure is different in that state.

This has clear precedent in Lorentz-type and later Lorentzian-pedagogy accounts of rods and clocks, where contraction and dilation are treated as consequences of the dynamical laws governing moving matter rather than as primitive geometric postulates (Bell 1987; Brown 2005). M1 adopts the same constructive instinct, but expresses it in momentum-first terms: clock slowing and contraction are read as changes in physical cycling and material equilibrium produced by directed momentum expression.

2.6.4 A true frame

If time is a measure abstracted from physical change, then a frame is not a separate version of reality. It is a way of describing relations among physical systems. Different inertial frames may remain equally usable descriptions, and the inertial equations above do not identify a preferred frame. The mathematics of the overlap regime does not need to care.

The foundational picture is different. M1 describes one physical stage populated by momentum-bearing systems. If those systems have one actual configuration of change, then there is one actual present. Two observers may assign different simultaneity slices, but those slices cannot both be equally fundamental versions of now.

The true-frame claim should be read in that sense. It is not an additional equation derived in this subsection, and it is not a claim that the present formalism already tells us how to locate the frame. It is an interpretive consequence of the momentum/space ontology: frame descriptions are plural, but the underlying physical now is singular.

The next question is how the inertial baseline can be modified in controlled ways. That is the role of kinematic modifiers.

2.7 Kinematic Modifiers

Foundations has described the inertial baseline,

$$M = \sqrt{p_f^2 + p^2}.$$

In that baseline, a particle's momentum configuration and its motion are related in the familiar inertial way. A structured physical setting can change that relation without replacing the particle's momentum content or adding a new momentum substance.

M1 calls such an effect a **kinematic modifier**:

A kinematic modifier is a physical structural effect that changes how a particle's momentum configuration produces motion.

The term names a job, not a universal equation. A modifier is not assumed to be an extra force, a correction attached to an otherwise complete trajectory, or a multiplier applied to every momentum quantity. Each later engine must identify the relevant physical structure and derive the particular momentum-to-motion relation it produces.

2.7.1 What a modifier changes

A particle can retain its fermic and bosic momentum content while the physical expression of that content changes. The same directed bosic momentum may then produce a different translation rate, or the same bound material system may sustain a different internal motion and equilibrium.

Clocks and rulers can reveal such a change because they are moving physical systems. A clock sustains a repeatable internal cycle. A ruler sustains a bound equilibrium through the motions and interactions of its constituents. They are probes of a kinematic modifier, not separate modifier classes.

The defining test remains motion: what does the particle's momentum configuration physically produce under the structural condition being considered?

2.7.2 Gravity and expansion

Gravity and expansion both give the concept a concrete role, but they need not realize it through the same mechanism.

In gravity, organized momentum changes the local stage. That structured stage changes the conditions under which particle momentum becomes local cycling and translation. The gravity engine must derive the corresponding local motion, material, and light responses from a reciprocal particle-stage construction.

In homogeneous expansion, the stage state χ dresses the bosic slot of the core-momentum operator. The resulting translation-yield law is

$$\dot{x}_k = \chi^2 c \frac{p_k}{M_\chi}.$$

Here the modifier's mechanism is explicit: bosic-slot stage dressing changes how directed momentum produces displacement. Calling gravity and expansion kinematic modifiers identifies their shared physical role; it does not claim that their realization laws are identical.

2.7.3 Possible state-space effects

Some future effects may change motion by restricting the joint configurations available to particles rather than by adding a force to an already available trajectory. Exclusion may eventually admit such a reading.

That possibility remains open. Foundations does not yet classify exclusion as an established modifier mechanism, because no current M1 derivation shows how its many-particle state restriction descends from the same kind of structure used in gravity or expansion.

2.7.4 Relation to space-mediated momentum coupling

Space-mediated momentum coupling and kinematic modification answer different questions. SMC asks how momentum-bearing systems become dynamically related through space. A kinematic modifier asks how physical structure changes the motion produced by a particle's momentum configuration.

A coupling may create or sustain the structure that acts as a modifier. The concepts can therefore meet in one engine, but neither is a subtype of the other.

2.8 What Foundations Establishes

Foundations has given M1 a working inertial grammar: the primitive momentum quantities, the directional shell-reading conservation structure, the exact correspondence back to familiar relativistic variables, and the baseline relation

$$M = \sqrt{p_f^2 + p^2}.$$

That grammar is not the whole theory. It is the language in which the rest of the theory can be built.

2.8.1 The inertial foundation

The opening sections fixed the core terms and assembled them into a physical momentum configuration. Fermic momentum p_f names the locked intrinsic scale of the particle's internal cycle. Botic momentum p belongs to that same spherical cycle. It has a spherical contribution and carries an asymmetry with a net direction; its signed component along a chosen orientation $\hat{k}$ is p_k . In flat space the fermic and botic roles are perpendicular contributions to one configuration, and the core momentum M measures its complete momentum content.

ADMC then gave that vocabulary its conservation structure. For an oriented unit direction $\hat{k}$, the particle's two positive shell readings are

$$p_k^\pm = M \pm \frac{1}{2}p_k.$$

ADMC conserves

$$\sum_i p_{k,i}^+$$

for every chosen orientation. Reversing $\hat{k}$ exchanges p_k^+ and p_k^- . Their average recovers M , while their difference recovers signed p_k . Along three independent spatial directions, the resulting four quantities are exactly and invertibly related to the familiar special-relativistic four-momentum package.

The point is not that M1 has abandoned standard inertial relativity. The point is that it has reorganized the same successful structure around momentum-first primitives and a foundational shell-reading conservation statement.

2.8.2 Space, time, and physical change

Foundations also fixed the framework's first interpretive order. Space is the stage. Momentum is the actor of physical change. A particle is one stable organization of momentum, but momentum does not have to be carried by a particle: the physical structure of space can, in certain situations, hold momentum and participate in its transfer. Time is not introduced as a separate substance that drives evolution; it is a measure abstracted from stable physical change.

That stance gives the clock discussion its meaning. A clock is a physical system with a stable cycle. In M1, clock slowing is read as changed physical cycling, not as time itself stretching. In the inertial case, the standard relation is recovered through

$$d\tau = dt \frac{p_f}{M},$$

with p_f/M playing the role of the usual inverse gamma factor. The interpretation is different, but the overlap relation is the familiar one.

The same stance also clarifies the true-frame language. Frame descriptions can be plural and operationally usable, but the underlying physical now is singular in the momentum/space ontology. That is an interpretive commitment of the foundation, not an additional inertial equation.

2.8.3 Departures from the inertial baseline

The final foundational move is to make room for controlled departures from the inertial case. The baseline map remains

$$M = \sqrt{p_f^2 + p^2},$$

but it is not assumed to exhaust every physical setting. A structured stage can change how a particle's momentum configuration produces motion without replacing that momentum content.

M1 calls such a structural effect a **kinematic modifier**. The term identifies a shared physical role, not a universal multiplier or a single mechanism. Gravity and expansion may both modify the relation between momentum and motion, while deriving that change in different ways. In expansion, basic-slot stage dressing supplies a specific translation-yield law. In gravity, the local stage structure must supply its own particle, material, and light realization.

Clocks and rulers can reveal a modifier because they are physical systems sustained by internal motion and material equilibrium. They are probes, not separate modifier classes. Possible exclusion physics remains a later question: it may eventually restrict the motions available to a many-particle configuration, but Foundations does not yet treat that possibility as an established modifier mechanism.

Space-mediated momentum coupling gives the interaction side of the picture. It states that momentum-bearing systems become dynamically related through space. Kinematic modifiers give a structural motion side: they identify settings in which physical structure changes what motion a particle's momentum produces.

This is room for later engines, not a completed engine in Foundations. ADMC remains the conservation grammar. SMC supplies the general coupling principle. Each kinematic modifier must still be given a concrete mechanism and realization law where it is used.

2.8.4 What remains beyond Foundations

Foundations has not built the gravity program, the quantum program, the cosmology program, or the later state-space and composite-structure programs. It has not derived full field dynamics, internal particle geometry, or final observational tests. Those are downstream tasks.

This boundary is intentional. A foundation should make the later theory legible before it tries to make the later theory powerful. The reader now has the primitive quantities, the conservation structure, the inertial correspondence, the stage/actor interpretation, the possibility of momentum held directly by space, the clock and true-frame stance, and the kinematic-modifier concept needed to recognize a structurally changed momentum-to-motion relation.

The next part begins with gravity because gravity is the first major case where organized momentum changes the local stage and thereby changes the conditions under which particle momentum becomes motion.

Part II

Tier 2 — Engines

3 Gravity and Structured Spacetime

3.1 Gravity as a Deformation of Space

A beam of light passing a massive body leaves in a different direction. That turn is also a change of momentum. For M1, the explanation must therefore reach beyond the beam's path to the system that turns it: the source, the surrounding space, and the momentum exchanged between them.

At the core of M1 lies **Additive Directional Momentum Conservation (ADMC)**. In an isolated system, the positive momentum contributions must add to an unchanged total along any direction. Gravity must meet this requirement while accounting for falling bodies, bent light and altered clock rates. How can momentum produce all these changes without breaking its own conservation rule?

The starting claim is that momentum deforms the geometry of the space it occupies, much as a vibration can stretch a string. Changes in that deformation propagate through space, and particles elsewhere respond to the resulting deformation at their own locations. M1 distinguishes this propagation of spatial deformation from the propagation of particles through space. It proposes that space itself does not undergo the slowing of internal processes experienced by material clocks, and that an event horizon need not prevent spatial disturbances from propagating outward. Establishing this proposed distinction requires a propagation law that permits it; the gravitational equations retained in this chapter do not yet establish that exception.

A particle's actual momentum configuration and propagation in those local spatial conditions constitute its **realization**. Of particular importance is the dilation of the geometry associated with a particle's mass. M1 proposes two linked effects. First, the dilated fermic resonant cycle requires less momentum, allowing momentum to relocate into the bosonic structure and participate in translation. In this interpretation, gravity can change a particle's motion without requiring a transfer of core momentum into the particle. Second, the particle's internal processes run more slowly when compared against a common reference. This is the proposed physical basis of gravitational time dilation. The chapter develops the corresponding motion and clock relations; deriving the internal redistribution from a resolved particle model remains a further task.

Figure 3.1 illustrates the process. Momentum deforms space, that deformation propagates, and observers are subjected to the resulting deformed space at their location. The sources

will be described from the bottom up, from particle contributions to the M1 version of the Stress-Energy tensor. Field variables describe the resulting deformation and its propagation. The particle response then connects that deformation to clock rates, acceleration, movement and light deflection.

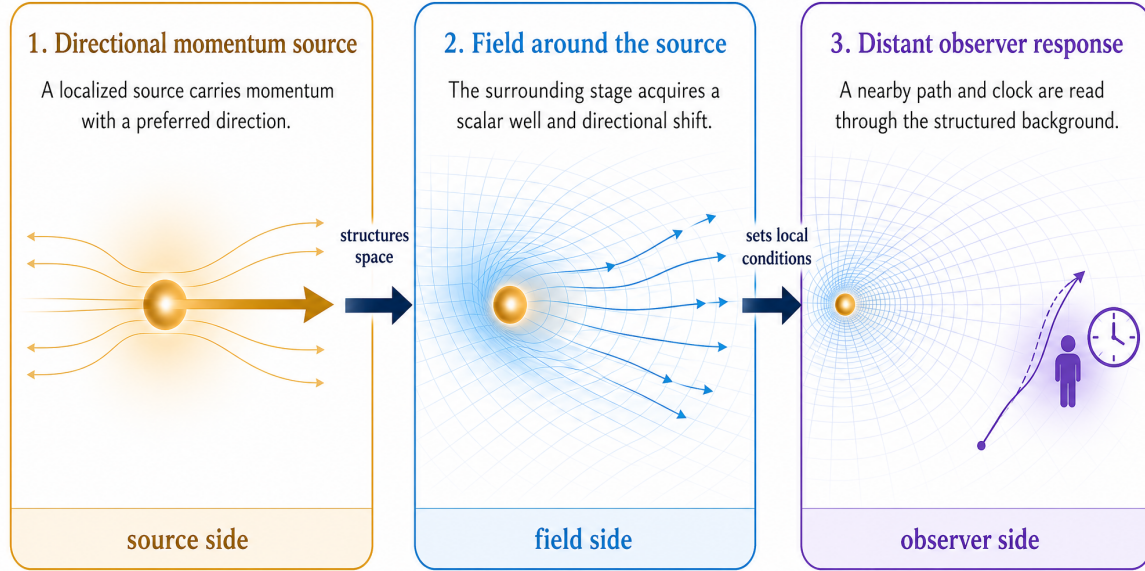


Figure 3.1: From momentum source to gravitational response. The source structures the surrounding space; paths and clocks probe the altered local conditions. The one-way sequence separates source, deformation and response. A complete interaction must also account for momentum exchanged with the source and the intervening space.

The deformation acts as a **kinematic modifier**: a physical structural effect that changes how a particle's momentum configuration produces motion. Gravity is the first such modifier explored in M1; later chapters will also explore spatial expansion and the Pauli exclusion principle.

Now, light has no massive internal cycle to dilate, yet it still crosses the deformed space. Its local speed remains c while its direction can change along the path. Explaining gravity therefore requires both the internal response of massive particles and propagation through the surrounding geometry.

The interaction must also return to the source. The particles that respond to gravity also contribute to gravity themselves. As spatial dilation reduces the momentum required by their fermic cycles, momentum relocates into their bosonic structure and participates in translation. Their gravitational contribution must therefore be evaluated from this changed momentum configuration. Evolving spatial deformation also participates in the momentum balance. Its

contribution can be negligible in most approximations. Source, space and responding particles must therefore participate in one reciprocal account. A prescribed background can be useful for calculating a trajectory, but the complete conservation test requires the momentum changes on both sides.

This chapter develops a model of that interaction and follows it through to clock comparisons, massive motion and light bending. The directional conservation algebra specifies the required balance; completing that account for a full curved gravitational system remains open. The source and response laws supply concrete consequences to compare with Newtonian gravity and general relativity.

3.2 Core Terms and Variables

The notation distinguishes sources, spatial deformation, the map between local and reference descriptions, and the particles realized in that setting. Each domain has its own table. Within a table, the inputs come before the quantities built from them, with related components kept together. The label a identifies one particle, also called a carrier.

3.2.1 Gravitational sources

Symbol	Name	Definition or role
Individual particles		
M_a	Realized carrier core momentum	Positive local core of particle a , including its internal and translational contributions.
p_a^i	Realized translation	Local spatial momentum, with $p_{i,a} = \delta_{ij} p_a^j$.
$p_{k,a}$	Signed directional projection	$p_{k,a} = \mathbf{p}_a \cdot \hat{k}$ for an oriented unit direction $\hat{k}$.
$p_{k,a}^\pm$	Opposed carrier readings	$p_{k,a}^\pm = M_a \pm p_{k,a}/2$.
Source construction		
$j_a^{\mu\nu}$	Single-carrier source kernel	$j_a^{\mu\nu} = p_a^\mu p_a^\nu / M_a$, formed before aggregation.
$\langle \cdot \rangle$	Local source moment	Sum with nonnegative weights per physical volume in the declared source frame.
$\mathcal{J}_k^\pm$	Opposed source channels	$\mathcal{J}_k^\pm = \langle \sum_a p_{k,a}^\pm \rangle$.
$J^{\mu\nu}$	Local momentum source tensor	Actual momentum content in a region of space, $J^{\mu\nu} = \langle \sum_a j_a^{\mu\nu} \rangle$.

Symbol	Name	Definition or role
Content, direction and transport		
$\mathcal{M}$	Additive source content	$\mathcal{M} = \langle \sum_a M_a \rangle = J^{00}$.
$\mathcal{P}^i$	Directional source imbalance	$\mathcal{P}^i = \langle \sum_a p_a^i \rangle = J^{i0}$.
$\mathcal{C}^{ij}$	Momentum transport	$\mathcal{C}^{ij} = \langle \sum_a p_a^i p_a^j / M_a \rangle = J^{ij}$.

Here $p_a^\mu = (M_a, p_a^i)$ collects the local momentum components, and $p_{\mu,a} = (-M_a, p_{i,a})$ lowers the index with the local signature. Lowercase j records one carrier; uppercase J records the aggregate. The calligraphic symbols distinguish population quantities from individual momenta. The blocks $\mathcal{M}$, $\mathcal{P}^i$ and $\mathcal{C}^{ij}$ belong to one source tensor. Source and deformation use all-upper components; the coframe remains a mixed-index map.

The source moment is taken at one event, in one declared frame. It is not a direct sum of components at separated locations. The particle construction in §3.3 supplies the kinetic case. Internal transport, binding and supporting stresses need their own consistent contributions to the same source; they are not fixed by the free-particle sum alone.

3.2.2 Spatial deformation

Symbol	Name	Definition or role
$\theta^{\mu\nu}$	Deformation tensor	Collects the common, directional and spatial stretch/shear components.
θ^{00}	Common deformation component	Core-content response; sourced by $\mathcal{M}$ in the weak-field equations.
θ^{0i}	Directional deformation components	Temporal–spatial entries, with $\theta^{i0} = \theta^{0i}$.
θ^{ij}	Spatial stretch/shear block	Spatial components of the same deformation array.

The array θ describes deformation relative to a specified reference. Its primary all-upper form is symmetric; reference-metric lowering is a separate operation.

3.2.3 Local and reference geometry

Symbol	Name	Definition or role
Conversion map		

Symbol	Name	Definition or role
$\phi^\mu{}_\nu$	Operational map	Converts coordinate increments dx^ν into local increments $d\ell^\mu$.
$(\phi^{-1})^\mu{}_\nu$	Inverse operational map	Converts local increments back into coordinate increments.
Components of the map		
$\phi^0{}_0$	Temporal map component	Temporal conversion; weakly $\phi^0{}_0 \simeq 1 - \theta^{00} - \delta_{kl}\theta^{kl}$. Distinct from a physical clock's cycle rate.
$\phi^i{}_0$	Spatial-temporal map components	Local spatial increments contributed by a coordinate-time increment.
$\phi^i{}_j$	Spatial map components	Convert coordinate spatial increments into local spatial increments.
Local increments and interval		
$d\ell^\mu$	Local temporal and spatial increments	$d\ell^\mu = \phi^\mu{}_\nu dx^\nu$; the temporal increment in time units is $d\ell^0/c$.
$g_{\mu\nu}$	Metric shadow	Interval representation derived from ϕ ; $g^{\mu\nu}$ is its inverse.

The map connects reference displacements to local geometry:

$$d\ell^\mu = \phi^\mu{}_\nu dx^\nu, \quad g_{\mu\nu} = \eta_{\alpha\beta} \phi^\alpha{}_\mu \phi^\beta{}_\nu. \quad (3.1)$$

The upper slot of ϕ is local output and the lower slot is coordinate input; the inverse reverses that direction. Its components belong to one map, and g is derived from it, not independently chosen. The Stage-adapted form used below has $\phi^0{}_i = 0$. A material clock or ruler still needs a model of the motion or equilibrium that produces its readings.

3.2.4 Particle realization and internal cycles

Symbol	Name	Definition or role
Particle identity and location		
p_f	Intrinsic fermic momentum	Fixed identity scale; written $p_{f,a}$ when identifying a particular particle.
x_a^μ	Carrier event / trajectory	Position in the declared coordinates, where the deformation is evaluated; $x^0 = ct$.
u_a^μ	Proper-time four-velocity	$u_a^\mu = dx_a^\mu/d\tau_a$, with $g_{\mu\nu}u_a^\mu u_a^\nu = -c^2$.
Realized momentum configuration		
p_a^i	Realized translation	Local spatial momentum, with $p_{i,a} = \delta_{ij}p_a^j$.
M_a	Realized carrier core momentum	Positive local core; the free-carrier approximation uses $M_a = \sqrt{p_{f,a}^2 + p_a^i p_{i,a}}$.
$p_a^\mu, p_{\mu,a}$	Carrier four-momentum	One momentum family in the declared frame; local $p_a^0 = M_a$.

Reference shell (static, zero shift) | | |

p_r^μ | Reference-expressed four-momentum | $p_r^\mu = \phi^0_0(p^\mu)_{\text{local}}$, in the map's local output axes with the common distant normalization. |

$M_r = -p_0$ | Reference core | $M_r = p_r^0 = \phi^0_0 M$; p_0 is the temporal coordinate covector component. |

p_r | Reference-expressed translation | $p_r = \sqrt{\delta_{ij}p_r^i p_r^j}$, the magnitude of the reference-expressed spatial momentum. |

$p_{k,r}$ | Signed reference projection | $p_{k,r} = \delta_{ij}k^i p_r^j$ for a unit direction in the same local output axes. |

$p_{k,r}^\pm$ | Opposed reference-shell momenta | $p_{k,r}^\pm = M_r \pm p_{k,r}/2$. |

$p_{f,r}$ | Reference-expressed fermic momentum | $p_{f,r} = \sqrt{M_r^2 - p_r^2} = \phi^0_0 p_f$; the local free-carrier shell retains p_f as its coefficient. |

Realization describes the particle's actual momentum configuration and propagation in the local deformation. The free-carrier shell used below keeps the free-space reference p_f fixed and combines it with translation. Resolving a particle's internal momentum and stresses requires its structural model. For nonzero lightlike carriers, $p_f = 0$ and $M = |\mathbf{p}| > 0$.

3.2.5 Indices, signs and units

Greek indices run over 0, 1, 2, 3; Latin spatial indices run over 1, 2, 3. A repeated tensor index is summed once up and once down. The labels f , r , a and k instead mark the fermic slot, its reference expression, a carrier and a reading direction. Carriers are summed explicitly with $\sum_a$; the label k in $p_{k,a}^\pm$ is not summed.

Frames are stated where quantities are used. Local source components use $\eta_{\mu\nu} = \text{diag}(-1, 1, 1, 1)$ and spatial components use δ_{ij} ; coordinate components use $g_{\mu\nu}$. This signature reverses the overall interval convention used in Foundations' inertial correspondence without changing its positive momentum readings. Ordinary indices are retained in both frames, and ϕ connects them. In ϕ^i_j , the upper and lower slots therefore belong to different bases despite both carrying spatial indices.

The individual quantities p_f , $p_{f,r}$, M , M_r , p_μ , p_r^μ and $j_a^{\mu\nu}$ have momentum units; u^μ has velocity units. Source densities such as J , $\mathcal{M}$, $\mathcal{P}$ and $\mathcal{C}$ have momentum-per-volume units. The deformation coefficients θ , the map coefficients ϕ , and the metric coefficients g are dimensionless in the chosen length coordinates. The conventional gravity dictionary is reserved for §3.10.

3.3 Gravitational Sources

In M1, momentum deforms space by creating tension in the geometry of space. To describe this tension or the resulting deformation, we must first establish a description of the amount and state of momentum in space. The deformation caused by momentum is naturally anisotropic, and the description must respect this. A natural first step is therefore to describe the momentum content of a volume through six directional channels, in a similar fashion to the momentum components of a particle's momentum shell. We denote these channels by $\mathcal{J}_k^\pm$, where k labels an arbitrary reading direction. These provide a primary measure of how much momentum occupies a region of space and how it is directed.

We begin in the weak-field approximation, treating the source particles as having their flat-space momentum configurations. Their gravitationally altered configurations, needed for stronger-field sources, come later, once the particle response to deformed space has been introduced in §3.5.

3.3.1 Adding the directional shells

Choose three perpendicular axes at the source point. Their opposed directions give six readings of the same local momentum content. Along any oriented direction $\hat{k}$, each particle supplies the familiar readings $p_{k,a}^\pm = M_a \pm p_{k,a}/2$, with $M_a = \sqrt{p_{f,a}^2 + p_a^i p_{i,a}}$. Adding them gives the primary source channels

$$\mathcal{J}_k^\pm = \left\langle \sum_a p_{k,a}^\pm \right\rangle. \quad (3.2)$$

The brackets turn the sum into a local density: each particle population is weighted by its number per unit volume, in the same frame. Both channels receive positive contributions, including from light. Reversing a particle's motion exchanges its opposed readings; it does not subtract that particle from the source.

Figure 3.2 connects the shell readings to their source densities. The same particles contribute to both channels; the side panels show their accumulation for identical particles with the same orientation.

From momentum shells to directional sources

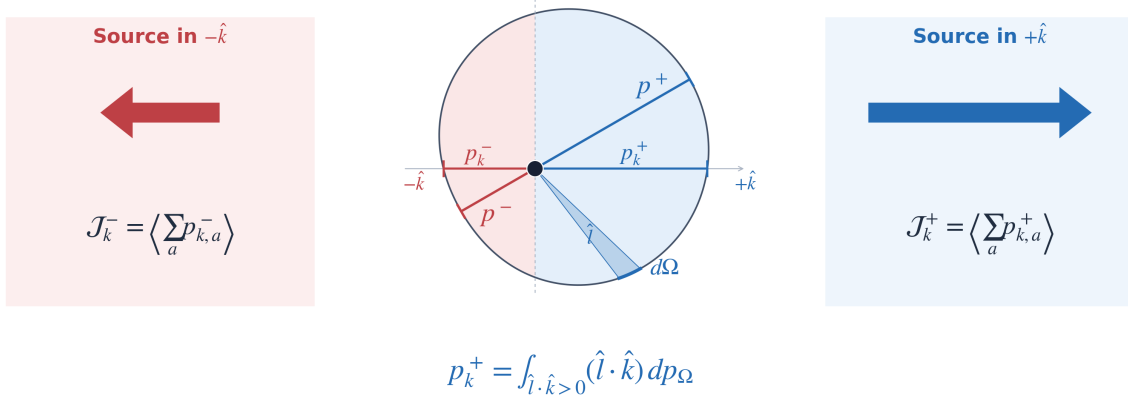


Figure 3.2: From momentum shells to directional source densities. The central contour plots one particle's shell readings; shading marks the opposed direction domains. Here $\hat{l}$ ranges over shell directions and $d\Omega$ is a solid-angle element. The displayed integral reconstructs p_k^+ with the auxiliary weight $dp_\Omega(\hat{l}) = \frac{1}{\pi} \left(M + \frac{3}{4} \mathbf{p} \cdot \hat{l} \right) d\Omega$. This weight represents the readings, not a microscopic momentum density.

To separate how much momentum is present from which way it is directed, take the mean and difference of the channels:

$$\begin{aligned} \mathcal{M} &= \frac{\mathcal{J}_k^+ + \mathcal{J}_k^-}{2} = \left\langle \sum_a M_a \right\rangle, \\ \mathcal{P}_k &= \mathcal{J}_k^+ - \mathcal{J}_k^- = \left\langle \sum_a p_{k,a} \right\rangle. \end{aligned} \quad (3.3)$$

The mean $\mathcal{M}$ is the same for every reading direction: it records the source's additive core content. The difference $\mathcal{P}_k$ records its directional imbalance and changes sign when the reading direction is reversed. Taking three spatial basis directions gives the components $\mathcal{P}^i = \langle \sum_a p_a^i \rangle$.

A rotating body can have no net translational momentum and still have a directional source pattern. Figure 3.3 makes that pattern visible. Take the reading direction $\hat{k}$ to the right. Particles in the upper half move against it, while particles in the lower half move along it. Their positive-channel contributions $\mathcal{J}_k^+$ are consequently smaller above and larger below.

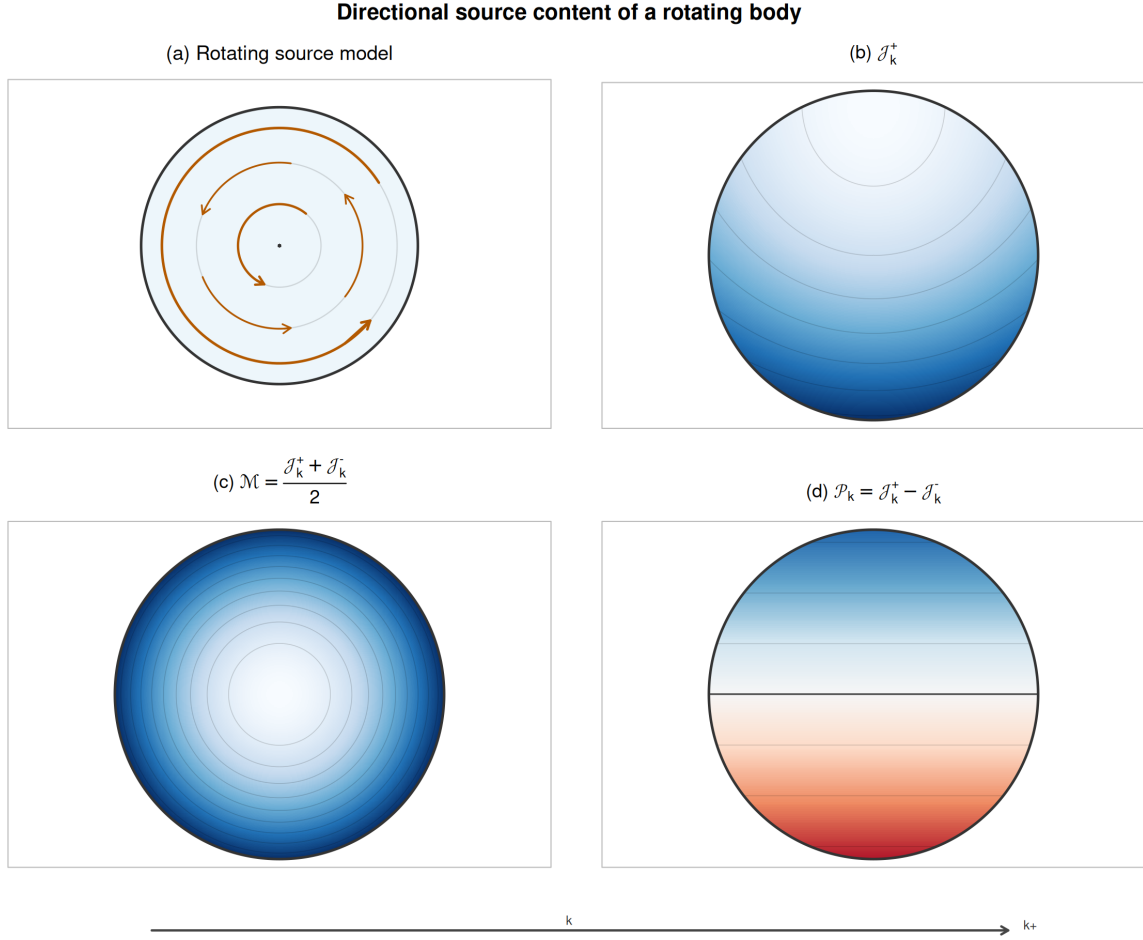


Figure 3.3: Directional source content in an illustrative rotating body. The particle number density is uniform, while tangential momentum increases with distance from the center. The mean $\mathcal{M}$ retains the core content; the difference $\mathcal{P}_k$ retains its directional imbalance. Blue and red in the difference panel mark motion against and along $\hat{k}$. These are source maps, not plots of the resulting deformation; each panel uses its own color scale.

The channel mean in panel (c) removes that directional sign. Its circular contours reflect the larger core momentum of the faster-moving particles farther from the center. The difference in panel (d) instead changes sign across the body. Summing that difference over the whole body would erase precisely the spatial pattern that distinguishes rotation from rest.

3.3.2 Pressure and shear

Particles also carry momentum across space. This adds a secondary part of the source description: the pressure and shear associated with their motion. The transport tensor is

$$\mathcal{C}^{ij} = \left\langle \sum_a \frac{p_a^i p_a^j}{M_a} \right\rangle. \quad (3.4)$$

The factor p_a^i/M_a is the particle's velocity in direction i in units of c . Multiplying it by p_a^j records the transport of momentum component j in that direction. For a population with no net local momentum, the diagonal entries give the kinetic normal stresses and the off-diagonal entries give the shear stresses. Equal diagonal entries describe isotropic pressure, expressed here in momentum-per-volume units.

This contribution comes from the same particle shells, but it retains information about how their individual momentum components occur together. The products must be formed particle by particle before averaging; they cannot generally be recovered from the summed channels alone.

For example, two equal streams, each with number density w , core M and momentum $\pm p$ along the first spatial axis give $\mathcal{J}_k^+ = \mathcal{J}_k^- = 2wM$ for every k , while $\mathcal{C}^{11} = 2wp^2/M$. Turn both streams onto the second axis: the primary source channels stay the same, but the transport moves to $\mathcal{C}^{22}$. The primary channels retain their content and net imbalance; the transport tensor distinguishes the opposing flow patterns.

3.3.3 The momentum source tensor

The field equations need content, directional imbalance and transport together. The source tensor $J^{\mu\nu}$ collects this actual momentum content in a region of space. For the particle population considered here, the local four-momentum $p_a^\mu = (M_a, p_a^i)$ gives the single-particle contribution and its aggregate:

$$j_a^{\mu\nu} = \frac{p_a^\mu p_a^\nu}{M_a}, \quad J^{\mu\nu} = \left\langle \sum_a j_a^{\mu\nu} \right\rangle = \begin{bmatrix} \mathcal{M} & \mathcal{P}^j \\ \mathcal{P}^i & \mathcal{C}^{ij} \end{bmatrix}. \quad (3.5)$$

The division by M_a makes the temporal entries recover the particle's core and spatial momentum, while the spatial block gives its momentum transport. Lowercase j_a records one particle's contribution; uppercase J records the region's momentum content. The all-upper array keeps $J^{00} = \mathcal{M}$ positive and its directional entries symmetric. The kinetic array is positive semidefinite, although signed directional and shear entries need not be positive. Derivation 3.3A gives the resolved construction and its frame-dependent weights; here we use its flat-space particle limit.

In the weak field, the core content $\mathcal{M}$ sources the common deformation θ^{00} . The transport trace also affects temporal readout, through the reconstruction of ϕ . The negative spacetime trace is a different combination. In the present source approximation,

$$-\eta_{\mu\nu}J^{\mu\nu} = \mathcal{M} - \delta_{ij}\mathcal{C}^{ij} = \left\langle \sum_a \frac{p_{f,a}^2}{M_a} \right\rangle. \quad (3.6)$$

Subtracting the transport trace isolates the fermic contribution in this weighted form. Cold particles at rest make both combinations equal to $\mathcal{M}$. Light has $M = |\mathbf{p}|$ and hence $\mathcal{M} = \delta_{ij}\mathcal{C}^{ij}$, while its temporal-readout combination is $2\mathcal{M}$. A light bath can therefore have zero net direction and zero spacetime trace without ceasing to gravitate. Secondary transport is not necessarily small.

These formulas give the kinetic particle contribution. Binding, walls and supports require their own stresses; they cannot all be replaced by a sum of free streams. Deformation can carry momentum too. A complete gravitational account must include each contribution once, with its source representation supplied by consistent dynamics.

3.4 The Deformation of Space

Space has an underlying geometry. The exact description of this geometry is not yet established, but string theory does resonate strongly with the momentum structures we see. Exploration of this geometry will be carried out in part 0. For now, we will limit ourselves to describing the deformations.

With a description of the source in place, the next step is to describe how this geometry is deformed by momentum, and then connect that deformation to local displacement and cycling.

3.4.1 From source channels to deformation

The source tensor $J^{\mu\nu}$ describes the actual momentum content in a region of space. A deformation can redistribute a particle's fermic and bosic content. Its resulting momentum configuration enters J without changing the source definition.

The source content, directional imbalance and momentum transport enter different components of the surrounding deformation. In the weak stationary field law, $\mathcal{M}$ contributes to the common deformation θ^{00} , while the local imbalance $\mathcal{P}^i$ sources the temporal-spatial components θ^{0i} . The secondary transport tensor $\mathcal{C}^{ij}$ supplies spatial stretch and shear, and its trace contributes to temporal readout through ϕ .

For a localized, conserved stationary leading source, with the deformation's own stress negligible at this order and the fields vanishing at infinity, the aligned harmonic-coordinate response is

$$\theta^{\mu\nu}(\mathbf{r}) = \frac{G_N}{c^3} \int \frac{J^{\mu\nu}(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d^3\mathbf{r}'. \quad (3.7)$$

Every source component supplies the corresponding deformation component with the same coupling. Expanding the source into the blocks defined in §3.3 gives

$$\begin{aligned} \theta^{00}(\mathbf{r}) &= \frac{G_N}{c^3} \int \frac{\mathcal{M}}{|\mathbf{r} - \mathbf{r}'|} d^3\mathbf{r}', \\ \theta^{0i}(\mathbf{r}) &= \frac{G_N}{c^3} \int \frac{\mathcal{P}^i}{|\mathbf{r} - \mathbf{r}'|} d^3\mathbf{r}', \\ \theta^{ij}(\mathbf{r}) &= \frac{G_N}{c^3} \int \frac{\mathcal{C}^{ij}}{|\mathbf{r} - \mathbf{r}'|} d^3\mathbf{r}'. \end{aligned} \quad (3.8)$$

Here G_N is the coupling of the adopted field law, and all source quantities are evaluated at $\mathbf{r}'$. The inverse-distance kernel carries their spatial pattern into the surrounding deformation. Reversing the rotation reverses $\mathcal{P}^i$ and its directional response, while leaving $\mathcal{M}$ and the kinetic transport unchanged. A stationary rotating body also needs the stresses that sustain its motion; these must be included in its source.

3.4.2 From deformation to local increments

The registered deformation determines the metric shadow used to construct local increments:

$$\begin{aligned} g_{00} &= -1 + 2(\theta^{00} + \delta_{kl}\theta^{kl}), \\ g_{0i} &= -4\delta_{ij}\theta^{0j}, \\ g_{ij} &= (1 + 2\theta^{00} - 2\delta_{kl}\theta^{kl})\delta_{ij} + 4\delta_{ik}\delta_{jl}\theta^{kl}. \end{aligned} \quad (3.9)$$

These components reconstruct the operational map ϕ . We fix the local spatial axes by taking $[\phi^i_j] = [g_{ij}]_+^{1/2}$, the symmetric positive-definite square root. Its spatial Gram matrix then reproduces g_{ij} , while the temporal and mixed blocks complete the same map. This frame choice leaves the sourced temporal-spatial components intact. We use the regular domain with a positive spatial metric and future temporal coefficient $\phi^0_0 > 0$.

With weak deformation in this frame, the map reduces to

$$\begin{aligned}\phi^0_0 &\simeq 1 - \theta^{00} - \delta_{kl}\theta^{kl}, & \phi^0_i &= 0, \\ \phi^i_0 &\simeq -4\theta^{0i}, \\ \phi^i_j &\simeq (1 + \theta^{00} - \delta_{kl}\theta^{kl})\delta^i_j + 2\delta_{jl}\theta^{il}.\end{aligned}\tag{3.10}$$

The temporal coefficient converts a coordinate-time increment into a local temporal increment. The mixed components contribute local spatial increments from that same coordinate-time increment; the spatial block converts coordinate displacements. In the registered convention $\theta^{10} = \theta^{01}$, so the corresponding directional map is $\phi^1_0 \simeq -4\theta^{10}$.

The linear response can be assembled one channel at a time. Take matched profiles in opposite directions, $\mathcal{J}^+_1$ and $\mathcal{J}^-_1$, with equal positive amounts Q^+ and Q^- :

$$Q^\pm = \int \mathcal{J}^\pm_1(\mathbf{x}) d^3\mathbf{x}, \quad Q^+ = Q^- > 0.\tag{3.11}$$

Both profiles use the axial shape $f(z) = e^{-|z|/r_0}/(2r_0)$, normalized to unit integral. Figure 3.4 compares the full positive-channel profile with half of each opposed profile. The respective amounts are $(Q^+, 0)$ and $(\frac{Q^+}{2}, \frac{Q^-}{2})$. Their sum is unchanged; their difference cancels. The figure isolates these channel contributions to the weak external map, keeping the transport contribution separate.

3.4.3 The sourced equations

For the external deformation, the model retains Einstein metric dynamics as an explicit physical input. This is a choice of field law, not a derivation of that law from ADMC. In native source notation,

$$G^{\mu\nu}[g(\phi)] = \frac{8\pi G_N}{c^3} (J^{\mu\nu})_{\text{coord}}.\tag{3.12}$$

Here $G^{\mu\nu}$ is the Einstein curvature operator of the metric shadow. Both indices of the displayed curvature operator are raised with the coordinate metric. The source is expressed in all-upper coordinate components: the conversion from the local source frame is $J_{\text{coord}} = \phi^{-1} J_{\text{local}} \phi^{-\text{T}}$. This changes the representation of the source, not its momentum content. The constant G_N belongs to this metric law. Each source contribution has momentum-per-volume units.

A solution also needs initial and boundary data. A changing configuration requires the initial deformation, its dynamical data and a compatible carrier population. A stationary exterior requires interior matching and whatever stresses maintain the source. The equations do not turn a prescribed body into a self-supporting one.

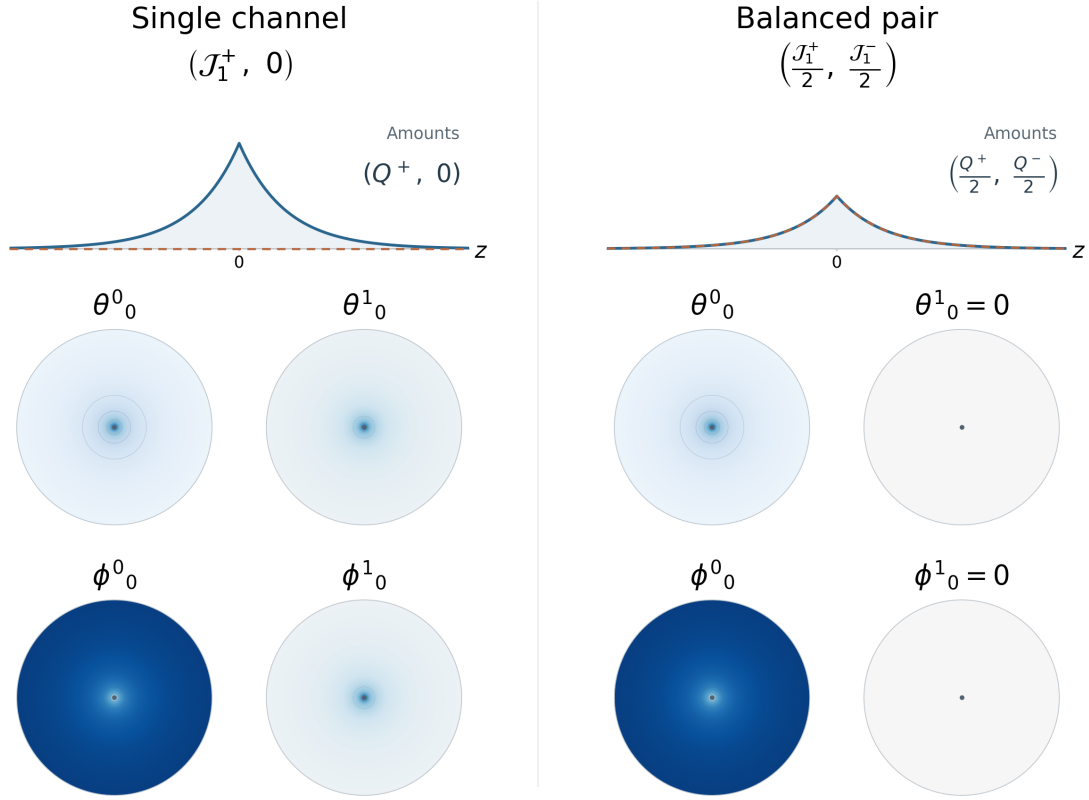


Figure 3.4: Primitive channel contributions. The pairs above the graphs name the whole source profiles; the amounts alongside are their integrals. A full positive-channel profile (left) is compared with half of each matched opposed profile (right). The temporal response remains while the directional response cancels. The ϕ panels use the first-order map.

Derivation 3.4A constructs the finite map and obtains the free-carrier source from the same action used for motion in §3.5. A resolved material source also needs the internal and supporting stresses of its own particle model.

3.4.4 A weak spherical source

A simple exterior makes the source-to-deformation step concrete. Take a spherical body of radius R_s , dominated by cold, slowly moving carriers with weak binding and small support corrections. Let M_{src} be its integrated leading core content and define the length coefficient

$$\ell_g = \frac{G_N M_{\text{src}}}{c^3}. \quad (3.13)$$

Normalize the distant reference by $\phi^0_0 \rightarrow 1$ and $\phi^i_j \rightarrow \delta^i_j$. Outside the body, at the retained weak order,

$$\begin{aligned} \phi^0_0(r) &\simeq 1 - \frac{\ell_g}{r}, \\ \phi^i_j(r) &\simeq \left(1 + \frac{\ell_g}{r}\right) \delta^i_j, \quad \phi^0_i = \phi^i_0 = 0. \end{aligned} \quad (3.14)$$

The temporal coefficient is smaller near the source, while the spatial map stretches coordinate displacements into local lengths. In this nonrotating, orientation-aligned example $\theta^{00} \simeq \ell_g/r$, with the other registered deformation components zero. The spatial map still changes because it depends on the full dictionary in Equation 3.9.

The approximation requires $\ell_g/R_s \ll 1$, small transport and binding/support corrections to the leading source, and observations in the exterior, with $r \gg R_s$. Interior matching and the stresses that maintain the body are supplied, not solved. Derivation 3.4A gives the weak exterior construction. The weak exterior motion and light comparisons retain these assumptions; effects smaller than the omitted source or nonlinear corrections are not resolved by them. The later finite-geometry and material examples state their own source laws and regimes.

3.5 Particles in Deformed Space

A particle entering deformed space responds through both its motion and its internal structure. Expansion of the internal fermionic geometry lowers its realized fermic momentum. As the particle falls freely through a stationary gravitational well, momentum redistributes between the fermionic and bosonic structures while its reference core remains conserved.

3.5.1 The directional momentum shell

A particle's four-momentum determines its entire directional momentum shell. The core fixes the shell's mean, while the basic momentum sets the difference between opposed directions.

Keep a static observer far from the well, where the map approaches the identity, and a static, zero-shift geometry with $\phi^0_i = \phi^i_0 = 0$ and $\phi^0_0 > 0$. Let p_f denote the particle's fermic momentum in its free-space realization, taken as the undeformed reference for this comparison.

For a massive free carrier, write its future-directed four-velocity as $u^\nu = dx^\nu/d\tau$, where τ is geometric proper time along its path. Its normalization is $g_{\mu\nu}u^\mu u^\nu = -c^2$. The map converts this coordinate four-velocity into local components. Expressing the corresponding momentum in the distant normalization gives

$$p_r^\mu[\phi, u] = \frac{p_f \phi^0_0}{c} \phi^\mu_\nu u^\nu. \quad (3.15)$$

The upper index of p_r^μ labels the fixed local axes of the map; the common factor ϕ^0_0 puts all components in the distant normalization. Extract the core and the signed basic projection along a unit spatial direction k , with $\delta_{ij}k^i k^j = 1$ in those same axes:

$$M_r = p_r^0, \quad p_{k,r} = \delta_{ij}k^i p_r^j. \quad (3.16)$$

In particular, $M_r = \phi^0_0 M = -p_0$, where M is the local core and p_0 is the coordinate covector component. The opposed shell momenta are

$$p_{k,r}^\pm = p_r^0 \pm \frac{1}{2} \delta_{ij} k^i p_r^j. \quad (3.17)$$

Substituting the four-momentum gives the shell directly from the deformation map and the particle's four-velocity:

$$p_{k,r}^\pm[\phi, u] = \frac{p_f \phi^0_0}{c} \left(\phi^0_\nu \pm \frac{1}{2} \delta_{ij} k^i \phi^j_\nu \right) u^\nu. \quad (3.18)$$

For the 2D drawing, take a plane containing the basic momentum and measure α from its direction. The full outline is the polar plot

$$R(\alpha) = M_r + \frac{p_r}{2} \cos \alpha, \quad p_r = \sqrt{\delta_{ij} p_r^i p_r^j}. \quad (3.19)$$

Here R is only the plotted momentum radius, not a new physical field. The forward and backward values are $M_r \pm p_r/2$, while every exactly perpendicular value is M_r . At rest, $p_r = 0$ and the outline is a circle.

The core circle has radius M_r . The fermic circle has radius

$$p_{f,r} = \sqrt{M_r^2 - p_r^2} = \phi_0^0 p_f. \quad (3.20)$$

These relations reconstruct the directional momentum shell and its two reference circles. They do not determine a unique microscopic momentum density or a physical particle boundary.

3.5.2 At rest in the well

Compare two realizations of the same particle: one at rest far from a static source, the other settled at rest in the well. With no translation, $p_r = 0$. The realized fermic and core momenta therefore coincide:

$$p_{f,r} = M_r = \phi_0^0 p_f = \phi_0^0 M. \quad (3.21)$$

Far away, where the map approaches the identity, the particle is realized with $M = p_f$. In the well, $\phi_0^0 < 1$ gives less realized fermic momentum and a smaller core in the same comparison.

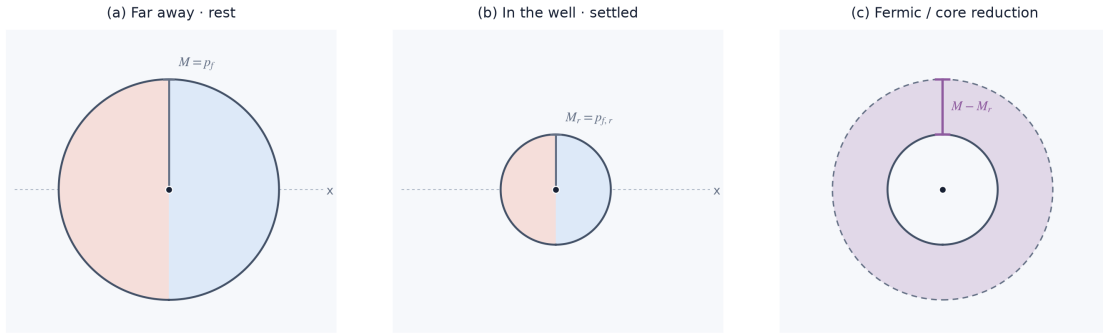


Figure 3.5: Resting states in a common reference. Panels (a) and (b) compare the distant and settled particles; (c) shows their difference. At rest the fermic and core momenta coincide. The well has $\phi_0^0 = 0.5$, and all panels share one momentum scale. The contours represent the momentum configuration, not the particle's spatial boundary.

The purple area shows the fermic reduction between these resting states.

3.5.3 Motion and directional momentum

A moving carrier also carries directed basic momentum. The fermic circle, core circle and full directional outline can therefore be distinguished.

Compare two such realizations at the same reference velocity. For translation along $+x$ with a diagonal spatial map,

$$\frac{v}{c} = \frac{\phi_0^0}{\phi_1^1} \frac{p_r}{M_r}. \quad (3.22)$$

A smaller temporal coefficient and a larger spatial coefficient reduce the velocity produced by a given basic-to-core ratio. To match the distant particle's reference velocity, the particle in the well therefore needs a larger ratio. In the example below, it has more reference-expressed basic momentum even though its reference core is smaller.

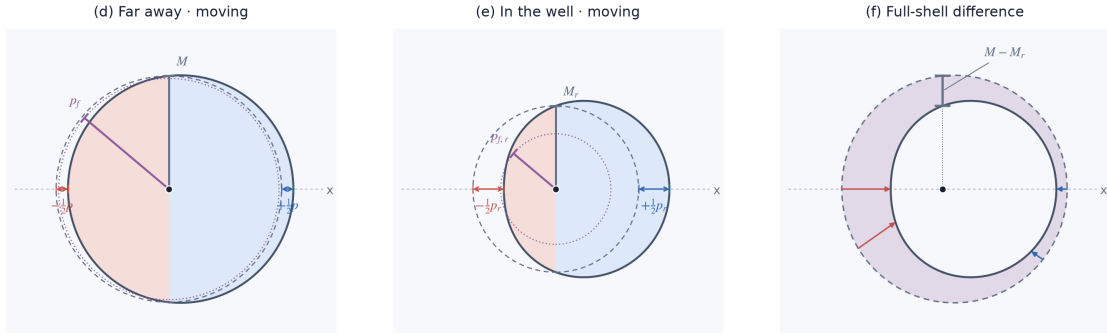


Figure 3.6: Moving states at the same distant-reference velocity, $v = 0.21c$. In (d) and (e), dotted circles mark fermic momentum, dashed circles mark core momentum, and solid outlines represent the directional shells. Panel (f) compares the full outlines; its vertical marker measures the core difference. Bare M, p refer to the distant state. The well uses $\phi_0^0 = 0.5$ and $\phi_1^1 = 16/9$. These are matched-motion snapshots, not stages of a fall or a circular orbit.

The unequal gaps around panel (f) show why the moving comparison is more than a uniform reduction. Perpendicular to the motion, the gap is $M - M_r$. Along other directions it also contains the change in basic imbalance.

3.5.4 Free fall, then settling

A freely falling particle changes its momentum configuration under a different condition: its reference core is conserved. Release it from rest in the distant limit. Stationarity fixes $M_r = p_f$ along the free trajectory, and the reference budget gives

$$M_r = p_f, \quad p_r = p_f \sqrt{1 - [\phi_0^0]^2}. \quad (3.23)$$

As the particle enters a deeper part of the well, $p_{f,r}$ decreases and p_r grows. The dashed core circle stays fixed while the directional outline changes. Its local core rises according to $M = M_r/\phi_0^0$; it is the common-reference core, not the local one, that is conserved.

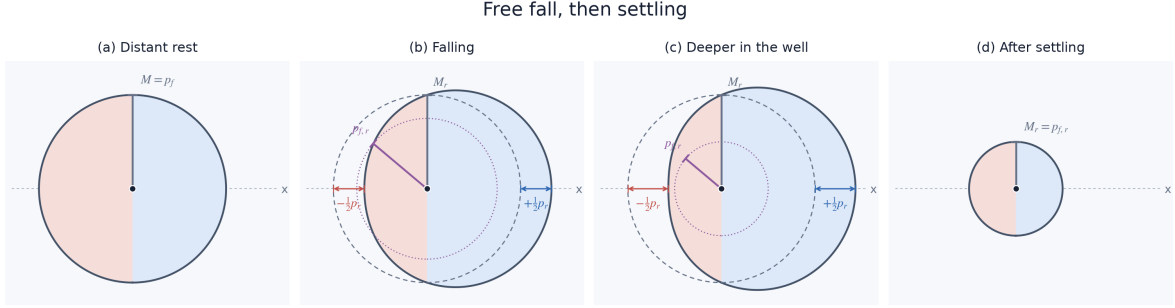


Figure 3.7: Free fall, then settling. Panels (a–c) use $\phi_0^0 = 1, 0.75, 0.5$: the reference core stays fixed while realized fermic momentum decreases and the bosic imbalance grows. Dotted circles mark fermic momentum, dashed circles mark core momentum, and solid outlines represent the directional shells, all on one momentum scale. Panel (d) shows a separate settling interaction at the last depth. The contours represent momentum configurations, not particle boundaries.

Settling removes the acquired translation. At the same depth, the final state has $p_r = 0$ and $M_r = p_{f,r}$. The difference $p_f - p_{f,r}$ is transferred to the surroundings. A particle settled in the well is therefore not the outcome of free fall alone: an interaction must remove core momentum as it stops the particle.

This example follows translational bosic momentum. In a structured particle, internal compensation need not all become centre-of-mass motion; its internal transport and supporting stresses require a structural model. Derivation 3.5A derives the free-carrier motion law, reference budget and stationary conservation.

3.5.5 Internal cycles and clocks

A clock counts a repeatable internal cycle. Let φ mark progress around a fermic cycle, with 2π corresponding to one recurrence. For the ideal clock used here, held at rest with its cycle undisturbed by the support, take the cycling rate in the common reference to be

$$\left(\frac{d\varphi}{dt}\right)_{\text{held}} = \frac{cp_{f,r}}{\hbar} = \frac{cp_f}{\hbar} \phi_0^0. \quad (3.24)$$

The realized fermic scale sets this resting rate. Motion adds the slowing factor established in Foundations: the ratio of fermic momentum to core momentum. In the common reference that ratio is $p_{f,r}/M_r = p_f/M$, so the moving cycle rate is

$$\frac{d\varphi}{dt} = \frac{cp_{f,r}}{\hbar} \frac{p_{f,r}}{M_r} = \frac{c}{\hbar} \frac{p_{f,r}^2}{M_r}. \quad (3.25)$$

The dotted fermic circle and dashed core circle now connect the momentum configuration to physical cycling. A lower fermic scale slows the resting cycle; a larger core relative to that scale slows it further through motion.

At a fixed depth, settling removes the translational slowing, while the gravitational difference from the distant resting clock remains. Applying this ideal-cycle comparison to a particular material clock requires the internal dynamics and support conditions that keep its cycle stable. Section 3.10 turns the rate into a counted-pulse experiment.

3.5.6 Light in deformed space

Light has no fermic internal cycle: $p_f = 0$. Its momentum shell has $M_r = p_r$ and $p_{f,r} = 0$. The local shell is $M = |\mathbf{p}|$, and the null momentum relation gives

$$-p_0 = \phi^0_0 \sqrt{g^{ij} p_i p_j}. \quad (3.26)$$

In stationary space this reference momentum is conserved along the ray. The local core can still vary, and the direction changes as the ray crosses the spatially varying map. Locally the speed is c ; a reference-coordinate path can nevertheless bend. Clock slowing and light bending therefore probe different aspects of the same deformation.

3.6 Reciprocal Sources

Every source particle exists in deformed space. A single particle will deform space locally, and its momentum configuration is, hence, never truly that of “free space”. To determine its actual momentum shell, we must account for both its motion and the deformation at its location. That shell supplies its contribution to J .

3.6.1 The source contribution of realized particles

Continue with the free carriers in §3.5's static, zero-shift geometry. Their momenta were expressed in a common distant reference, whereas the source formula of §3.3 uses local components. The conversion is $p_r^\mu = \phi^0_0 p_{\text{local}}^\mu$, with $M_r = \phi^0_0 M$. Expressing the particle source in these reference quantities gives

$$j_a^{\mu\nu} = \frac{p_{r,a}^\mu p_{r,a}^\nu}{\phi^0_0 M_{r,a}}, \quad J^{\mu\nu} = \left\langle \sum_a \frac{p_{r,a}^\mu p_{r,a}^\nu}{\phi^0_0 M_{r,a}} \right\rangle. \quad (3.27)$$

The brackets average the particle contributions over physical volume in the local frame. The factors of ϕ^0_0 convert the reference momenta back to the local components used by the source.

For a massive particle, $p_{r,a}^\mu$ follows from the deformation map and its motion, as derived in §3.5. The source formula is unchanged; the particles now contribute their momenta in deformed space.

3.6.2 The exact temporal feedback

Core momentum and momentum transport both source the temporal factor ϕ^0_0 . In the static geometry, the field law of §3.4 gives

$$D_i D^i \phi^0_0 = \frac{4\pi G_N}{c^3} \phi^0_0 (\mathcal{M} + \delta_{ij} \mathcal{C}^{ij}). \quad (3.28)$$

Here D_i is the spatial covariant derivative for g_{ij} , so the operator follows the deformed spatial geometry. The equation applies at finite field strength.

For the free-particle population, its source term can be written directly in the reference momenta:

$$\phi^0_0 (\mathcal{M} + \delta_{ij} \mathcal{C}^{ij}) = \left\langle \sum_a \left(M_{r,a} + \frac{\delta_{ij} p_{r,a}^i p_{r,a}^j}{M_{r,a}} \right) \right\rangle. \quad (3.29)$$

Both the core and the directed transport of §3.5's shell therefore enter the temporal field equation. For identical resting particles with local number density n , the kinetic terms reduce to $\mathcal{M} = np_f$ and $\mathcal{C}^{ij} = 0$, giving

$$D_i D^i \phi^0_0 = \frac{4\pi G_N}{c^3} np_f \phi^0_0 = \frac{4\pi G_N}{c^3} np_{f,r}. \quad (3.30)$$

The local fermic density is np_f ; the field equation weights it by ϕ^0_0 , giving $np_{f,r}$. Derivation 3.6A obtains this weighting from the full field equation.

3.6.3 A supported reciprocal solution

In a static sphere, a pressure gradient balances gravity, and the pressure also contributes to the source. A constant-density continuum gives one exact example. Let its total local core density be $\mathcal{M}_0$, with isotropic spatial stress. Define its radius R_s by the surface area $4\pi R_s^2$. The length $r_s = 8\pi G_N \mathcal{M}_0 R_s^3 / (3c^3)$ sets the strength of its exterior field. With vanishing stress at the surface, the full static equations give the interior temporal map

$$\phi^0_0(r) = \frac{1}{2} \left[3\sqrt{1 - \frac{r_s}{R_s}} - \sqrt{1 - \frac{r_s r^2}{R_s^3}} \right], \quad 0 \leq r \leq R_s. \quad (3.31)$$

Derivation 3.6A solves the spatial map and supporting pressure along with this temporal map. A positive central clock factor and finite central support require $r_s/R_s < 8/9$. The spatial deformation and the pressure that balances the body are parts of the same solution.

3.6.4 A simplified scalar feedback model

To isolate the source reduction, prescribe a uniform resting population inside a sphere of radius R_s , with n counted per Euclidean volume. As an additional scalar field-law postulate, extend the flat Poisson operator to finite deformation and source it with the reduced fermic content. Using $\phi^0_0 = \sqrt{1 - 2\theta^{00}}$ in this model gives

$$\frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{d\theta^{00}}{dr} \right) = -\frac{4\pi G_N n p_f}{c^3} \sqrt{1 - 2\theta^{00}}. \quad (3.32)$$

A regular centre and continuous matching to a decaying exterior determine the profile. For the dimensionless source strength $4\pi G_N n p_f R_s^2 / c^3 = 1$, the central and surface temporal factors are approximately 0.58138 and 0.74031, and the integrated source is 0.67791 of the unreduced value. The unexpanded square root makes the feedback explicit: increasing the well depth reduces the source that deepens it.

A related analytical example appears in Einstein's self-coupled scalar gravity, discussed by Giulini: a uniform sphere has a reduced active mass and a hyperbolic-function interior solution. Derivation 3.6A compares that model with the scalar postulate used here.

Particle motion changes the source, the source changes the geometry, and the geometry changes the subsequent motion and source again. Following this feedback in time requires the full evolution equations for the source and deformation.

3.7 Field Equations

A changing source changes the space through which it moves. The stationary source integral of §3.4 describes a weak field with no time dependence. To follow a travelling disturbance, or the effect of a finite deformation on its own evolution, we use the differential field law introduced in §3.4.

3.7.1 The finite field law

The deformation θ determines g and the local map ϕ through §3.4's relations. With the Einstein curvature operator chosen there, the field equation is

$$G^{\mu\nu}[g(\theta)] = \frac{8\pi G_N}{c^3} J_{\text{coord}}^{\mu\nu}, \quad J_{\text{coord}}^{\mu\nu} = (\phi^{-1})^\mu{}_\alpha (\phi^{-1})^\nu{}_\beta J^{\alpha\beta}. \quad (3.33)$$

The inverse map converts the local components of J into the coordinate components required by the field equation. On the left, $G^{\mu\nu}$ combines spatial and temporal variations of the geometry. Since g is determined by θ , this is a field equation for the registered deformation.

Derivatives of $g[\theta]$ first give the compatible connection, which relates local frames at neighboring points. Curvature then combines derivatives and products of that connection. The resulting operator contains second derivatives of θ , products of field gradients, and coefficients set by the finite geometry. Derivation 3.7A gives its explicit form and source normalization.

3.7.2 From the finite equation to propagation

For a weak deformation about the undeformed reference, choose coordinates satisfying $\partial_\mu \theta^{\mu\nu} = 0$ to leading order. The equation reduces to

$$\square \theta^{\mu\nu} = -\frac{4\pi G_N}{c^3} J^{\mu\nu}, \quad \square = -\partial_0^2 + \delta^{ij} \partial_i \partial_j. \quad (3.34)$$

Since $x^0 = ct$, the temporal and spatial derivatives describe propagation at c in the undeformed reference. Removing the time dependence gives §3.4's Poisson equation and its stationary source integral. Keeping it allows a source change to travel outward rather than alter the entire field instantaneously.

At finite deformation, the geometry determines the local propagation cone. The nonlinear terms also let one part of a gravitational disturbance influence another. The curvature operator already contains this gravitational self-coupling.

3.7.3 Initial data and constraints

To evolve a field, specify the geometry of a spatial slice, its rate of change, and the matter on it. The field equations link these initial data: their temporal projections constrain which combinations are possible. After the coordinates have been fixed, the remaining equations evolve the spatial geometry.

These constraints and the freedom to choose coordinates leave two local radiative polarizations among the ten symmetric components of θ . The fixed orientation of ϕ specifies the local axes, while the connection relates frames at neighboring points.

A supported body needs its material equations as well as these gravitational equations. A population of free particles needs its transport law and initial distribution. Supplying J from the same matter dynamics that moves the source is what turns a field solution into a reciprocal one.

3.7.4 Momentum balance through the geometry

The field equation requires the local momentum balance

$$\nabla_\mu J^{\mu\nu} = 0. \quad (3.35)$$

This balance uses the connection of the evolving geometry. In coordinate components it reads

$$\partial_\mu(\sqrt{-g} J^{\mu\nu}) = -\sqrt{-g} \Gamma^\nu_{\mu\alpha} J^{\mu\alpha}, \quad (3.36)$$

where $g = \det(g_{\mu\nu})$. The right-hand side accounts for the changing comparison of momentum components between neighboring events. Coordinate components can therefore have changing volume integrals even while this local balance holds.

ADMC adds a global requirement: for an isolated system, the positive opposed directional totals must remain conserved in a declared common comparison. In a flat common frame, closed core and signed-momentum balances, together with $\mathcal{M} \geq |\mathcal{P}|$, give

$$\int_V \mathcal{J}_k^\pm d^3x = \int_V \left(\mathcal{M} \pm \frac{1}{2} \hat{k} \cdot \mathcal{P} \right) d^3x. \quad (3.37)$$

Derivation 3.7A obtains these conserved positive totals for closed boundaries. In curved space, a common directional balance also needs a way to compare separated local axes and to include the momentum carried by deformation, support and boundary transport. Constructing that complete account remains open.

3.8 Time-Dependent Gravity

Moving particles change the momentum source, and a changing deformation alters their subsequent motion. When both evolve, gravity is no longer a sequence of stationary wells. The particle distribution and the geometry must advance together.

3.8.1 Momentum in a changing geometry

The free-particle law from §3.5 also applies to a changing geometry. In coordinate components it gives

$$\frac{dx^i}{dx^0} = \frac{p^i}{p^0}, \quad \frac{dp_\mu}{dx^0} = -\frac{1}{2p^0} \partial_\mu g^{\alpha\beta} p_\alpha p_\beta. \quad (3.38)$$

The second equation now includes changes in the temporal momentum:

$$\frac{d(-p_0)}{dx^0} = \frac{1}{2p^0} \partial_0 g^{\alpha\beta} p_\alpha p_\beta. \quad (3.39)$$

In a stationary geometry, $\partial_0 g^{\alpha\beta} = 0$ and $-p_0$ is conserved. Time dependence allows $-p_0$ to change according to the equation above. The conserved reference core in §3.5 followed from this time symmetry.

A collisionless population evolves by transporting particles through position and momentum space. Averaging their local contributions gives the changing J . The same transport law implies $\nabla_\mu J^{\mu\nu} = 0$, so the motion that updates the source also preserves its local momentum balance.

3.8.2 A disturbance travels through space

In the weak regime, with no incoming radiation, the sourced solution is retarded:

$$\theta^{\mu\nu}(t, \mathbf{x}) = \frac{G_N}{c^3} \int \frac{J^{\mu\nu}(t - |\mathbf{x} - \mathbf{x}'|/c, \mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|} d^3x'. \quad (3.40)$$

A change at the source reaches another point after a finite travel time. The stationary integral returns when the delay no longer changes the source values. Free gravitational radiation can be specified separately in the initial data.

In the weak vacuum, the equations admit two transverse wave polarizations. They change the tidal separation between neighboring free trajectories even when the common component θ^{00} vanishes. Describing these waves requires the spatial components of the deformation, beyond

a scalar well. At finite field strength, disturbances follow the local null cone of the evolving geometry.

Derivation 3.8A gives the transport and wave calculations. The full free-particle–field system has local evolution from regular, constraint-satisfying data under the conditions stated in Derivation 3.7A.

3.8.3 An exact coupled example

A uniform, isotropic population of light gives an exact coupled example. Choose a spatial map that stretches every direction by the same time-dependent factor:

$$\phi^0_0 = 1, \quad \phi^i_j = a(x^0)\delta^i_j, \quad \phi^0_i = \phi^i_0 = 0, \quad (3.41)$$

Here $a > 0$ sets the physical distance between coordinate points. Translation symmetry keeps each particle’s coordinate momentum p_i fixed, while its local spatial momentum is p_i/a . The number density per physical volume scales as a^{-3} : the same population occupies a changing volume.

Normalize $a(0) = 1$ and let $\mathcal{M}_0$ be the initial core density. Each nonzero light carrier has local core equal to its spatial momentum magnitude, so its core scales as a^{-1} . Together with the changing number density, this gives

$$\mathcal{M} = \mathcal{M}_0 a^{-4}, \quad \mathcal{P}^i = 0, \quad \mathcal{C}^{ij} = \frac{\mathcal{M}}{3} \delta^{ij}. \quad (3.42)$$

Solving the full field equations with this source gives the expanding solution

$$a^2(x^0) = 1 + 2\sqrt{\frac{8\pi G_N \mathcal{M}_0}{3c^3}} x^0. \quad (3.43)$$

The opposite sign before the time term gives the contracting solution. Both branches apply while $a > 0$. In each case, the particle trajectories, source and full field equations—constraints as well as evolution—are satisfied together.

The corresponding registered deformation is

$$\theta^{00} = \frac{3(a^2 - 1)}{8}, \quad \theta^{0i} = 0, \quad \theta^{ij} = \frac{1 - a^2}{8} \delta^{ij}. \quad (3.44)$$

The common component and spatial trace cancel in the temporal map, leaving $\phi^0_0 = 1$, while the spatial geometry evolves. Time derivatives now supply field terms that were absent from §3.6’s static equation. This example tests coupled gravitational evolution; the physical assumptions of the book’s separate expansion engine are introduced later.

3.8.4 A material recurrence can evolve too

An oscillating body provides a clock whose recurrence can be followed through changing geometry. Take a specified self-bound fluid model with a supported spherical equilibrium and a small radial oscillation. Finite perturbations make its surface move and its internal stresses evolve along with the gravitational field, so the period can shift from its linear value.

The numerical calculation follows two small but finite perturbations for six reference periods, with no signal probe present. At the larger amplitude, successive periods shorten slightly toward the linear value. The resolution study separates this change from numerical drift over the tested interval.

A time comparison must follow the moving surface as well. Its elapsed proper time is

$$\Delta\tau_s = \int_{t_1}^{t_2} \sqrt{-g_{\mu\nu}(x_s(t))} \frac{dx_s^\mu}{c dt} \frac{dx_s^\nu}{c dt} dt. \quad (3.45)$$

The integral accumulates gravitational and translational slowing along the actual surface motion. Both the geometry and the velocity must therefore be updated throughout the recurrence. This is the dynamical counterpart of §3.5's clock comparison, now applied to an evolving material body.

§3.10 compares this material recurrence and a separate weak scalar-signal calculation with GR. Finite material–signal exchange has been evolved in a homogeneous model; coupling such an exchange to the oscillating body and a distant observer remains open.

3.9 Gravity in Six Directional Components

The directional shell does not begin with one time slot and three spatial slots. It begins with six positive directional components: one for each branch of three opposed axes. This is the more M1-native way to display the same gravitational content. The ordinary four-component form remains available, but the six-component form shows source correlations, metric comparison and transport relations directly.

3.9.1 Six readings, four degrees of freedom

Choose one local orthonormal frame. Let a slashed component index $\ni$ label one of the six directional components,

$$\ni \in \{1+, 1-, 2+, 2-, 3+, 3-\}.$$

For a particle with core M and signed spatial components p_i , the six directional components are

$$p^{i\sigma} = M + \frac{\sigma}{2}p_i, \quad i = 1, 2, 3, \quad \sigma = \pm 1. \quad (3.46)$$

Equivalently, write these components as p^ϑ . The inverse relations are

$$M = \frac{1}{6} \sum_{\vartheta} p^\vartheta, \quad p_i = p^{i+} - p^{i-}. \quad (3.47)$$

Thus the six readings contain the ordinary core and signed momentum in a supported four-dimensional pattern. Their opposed pair sums satisfy

$$p^{1+} + p^{1-} = p^{2+} + p^{2-} = p^{3+} + p^{3-} = 2M. \quad (3.48)$$

The pair-sum equalities define a four-dimensional supported subspace inside the six displayed components. Future carriers also obey the mass-shell condition $M \geq |\mathbf{p}|$, so every directional component is positive. Using §3.5's reference-expressed momenta gives the same six-component shell for particles in deformed space.

For comparison with the ordinary form, order the six components as $(1+, 1-, 2+, 2-, 3+, 3-)$. Use L for both directions and let the indices state the direction of the map:

$$p^\vartheta = L^\vartheta_{\mu} p^\mu, \quad p^\mu = L^\mu_{\vartheta} p^\vartheta.$$

Here the ordered ordinary components are $p^\mu = (M, p_1, p_2, p_3)^\top$ and the ordered six-component list is $(p^{1+}, p^{1-}, p^{2+}, p^{2-}, p^{3+}, p^{3-})^\top$. In matrix form,

$$[L^\vartheta_{\mu}] = \begin{pmatrix} 1 & 1/2 & 0 & 0 \\ 1 & -1/2 & 0 & 0 \\ 1 & 0 & 1/2 & 0 \\ 1 & 0 & -1/2 & 0 \\ 1 & 0 & 0 & 1/2 \\ 1 & 0 & 0 & -1/2 \end{pmatrix}, \quad [L^\mu_{\vartheta}] = \begin{pmatrix} 1/6 & 1/6 & 1/6 & 1/6 & 1/6 & 1/6 \\ 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \end{pmatrix}. \quad (3.49)$$

Lifting and returning leaves the ordinary four-component vector unchanged:

$$L^\mu_{\vartheta} L^\vartheta_{\nu} = \delta^\mu_{\nu}.$$

The reverse contraction $L^\vartheta_{\mu} L^\mu_{\vartheta}$ returns the six readings to the equal-pair-sum support. Derivation 3.9A gives the projector and supported metric forms.

3.9.2 The metric compares common content with opposed differences

The same sums and differences that recover core and basic momentum also define the native reference metric. The core is the average of all six readings, while each basic component is an opposed difference. The shell invariant is therefore

$$\eta_{\ni j} p^{\ni} p^j = \sum_{k=1}^3 (p^{k+} - p^{k-})^2 - \left(\frac{1}{6} \sum_{\ni} p^{\ni} \right)^2 = -p_f^2. \quad (3.50)$$

This equation uses the local readings defined above. For the static reference readings $p_r^{\ni} = N p^{\ni}$, the same contraction is instead $-p_{f,r}^2 = -(N p_f)^2$.

The metric combines directional imbalance and common content into one invariant. The minus sign distinguishes those roles; it does not introduce negative momentum content. Component-wise, using the same branch notation as $p^{i\sigma}$, this is the supported reference metric:

$$\eta_{i\sigma, j\tau} = \sigma\tau \delta_{ij} - \frac{1}{36}. \quad (3.51)$$

The signs σ and τ are $+1$ on plus branches and -1 on minus branches. The Kronecker delta compares the two spatial axes; there is no hidden sum over the displayed component labels.

The deformed Stage applies the same structure to geometric increments. The readout map gives

$$d\ell^{\ni} = \phi^{\ni} dx^j. \quad (3.52)$$

The interval can then be written entirely in six-component language:

$$g_{\ni j} dx^{\ni} dx^j = \sum_{k=1}^3 (d\ell^{k+} - d\ell^{k-})^2 - \left(\frac{1}{6} \sum_{\ni} d\ell^{\ni} \right)^2. \quad (3.53)$$

The rule is the same as for the momentum shell: compare the three opposed displacement differences with the common process increment. Deformation changes that comparison through ϕ . These increments are geometric quantities, and in the retained correspondence their common increment supplies the local clock component. Equivalently,

$$g_{\ni j} = \eta_{kl} \phi^k_{\ni} \phi^l_j. \quad (3.54)$$

Source, shell and metric now share one organizing structure: common content plus opposed differences. The common mode carries the temporal role; the opposed differences carry the spatial roles. Metric coefficients can therefore have mixed signs even though the directed momentum readings themselves are positive.

3.9.3 Source correlations appear as component products

The source describes which directional components occur together. Form products for each carrier before averaging:

$$J^{\ni j} = \left\langle \sum_a \frac{p_a^{\ni} p_a^j}{M_a} \right\rangle. \quad (3.55)$$

The six-by-six source records correlations between directional components. The aggregate first moments recover $\mathcal{M}$ and $\mathcal{P}^i$, but the transport block $\mathcal{C}^{ij}$ depends on which directional readings occur together on the same carriers. Equal counterstreams along x and equal counterstreams along y can have the same core and zero net spatial momentum while transporting momentum in different directions.

For two equal carriers with momenta $\pm p\hat{x}$ and local core M , the opposed x components differ by

$$J_x^{1+,1+} - J_x^{1+,1-} = \frac{p^2}{M}, \quad J_x^{2+,2+} - J_x^{2+,2-} = 0. \quad (3.56)$$

For equal streams along y , the nonzero difference moves from the 1 pair to the 2 pair. The first moments are identical in the two cases; the component correlation shows which direction carries transport. Derivation 3.9A gives the calculation.

The ordinary source and the six-component source are connected by the same lift and recovery on both upper slots:

$$J^{\ni j} = L^{\ni}_{\mu} L^j_{\nu} J^{\mu\nu}, \quad J^{\mu\nu} = L^{\mu}_{\ni} L^{\nu}_{\jmath} J^{\ni j}. \quad (3.57)$$

The deformation has the same supported dictionary:

$$\theta^{\ni j} = L^{\ni}_{\mu} L^j_{\nu} \theta^{\mu\nu}, \quad \theta^{\mu\nu} = L^{\mu}_{\ni} L^{\nu}_{\jmath} \theta^{\ni j}. \quad (3.58)$$

The slashed components and the ordinary components are two views of the same supported object. A source written as $J^{\ni j}$ displays how the opposed directional readings correlate; applying $L^{\mu}_{\ni}$ to each slot returns the ordinary four-component source.

3.9.4 Curvature compares directional momentum between places

A source correlation asks which directional momenta occur together at one location. The geometric question is how the deformed Stage changes the comparison of those readings between neighboring locations. The connection supplies that comparison rule. Curvature measures its path dependence: transport the same momentum configuration along two infinitesimally different routes and compare the results.

In the fixed supported component basis,

$$[\nabla_{\vec{k}}, \nabla_{\vec{l}}]p^{\mathfrak{D}} = \mathcal{R}^{\mathfrak{D}}_{jkl}p^j. \quad (3.59)$$

The first two slots describe the momentum response; the last two describe the two transport directions. Curvature is an oriented change in transport, so reversing the path orientation reverses signs. The sign of a curvature component therefore records the orientation of comparison, not the presence of negative momentum content.

The field equation uses a particular two-index contraction of this transport curvature, the source-matched curvature response. In six-component form this response is the lifted two-index curvature combination

$$G^{\mathfrak{D}j} \equiv L^{\mathfrak{D}}_{\mu} L^j_{\nu} \left(R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R \right), \quad (3.60)$$

For this finite coordinate-curvature equation, convert the local source first:

$$J^{\mu\nu}_{\text{coord}} = (\phi^{-1})^{\mu}_{\alpha} (\phi^{-1})^{\nu}_{\beta} J^{\alpha\beta}_{\text{local}}, \quad J^{\mathfrak{D}j}_{\text{coord}} = L^{\mathfrak{D}}_{\mu} L^j_{\nu} J^{\mu\nu}_{\text{coord}}.$$

The retained field law is

$$L^{\mathfrak{D}}_{\mu} L^j_{\nu} \left(R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R \right) = \frac{8\pi G_N}{c^3} J^{\mathfrak{D}j}_{\text{coord}}. \quad (3.61)$$

Both sides now use coordinate components and the same supported six-component representation. The equation constrains the source-matched contraction of curvature. The full transport curvature $\mathcal{R}^{\mathfrak{D}}_{jkl}$ can still carry vacuum tidal structure and propagating disturbances where the local source term vanishes.

3.9.5 The weak solution component by component

In the aligned weak-harmonic limit,

$$G^{(1)\ni j} = -2\Box\theta^{\ni j}, \quad (3.62)$$

so the field equation becomes

$$\Box\theta^{\ni j} = -\frac{4\pi G_N}{c^3} J^{\ni j}. \quad (3.63)$$

For a localized conserved stationary source, this gives

$$\theta^{\ni j}(\mathbf{x}) = \frac{G_N}{c^3} \int \frac{J^{\ni j}(\mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|} d^3x'. \quad (3.64)$$

Each source correlation drives its corresponding deformation component in this weak limit. The finite operator then describes how all components jointly determine the comparison geometry through g , the connection and its curvature. Applying $L^\mu_{\ni}$ to both upper slots recovers the ordinary four-component equation and solution.

The six-component form brings the retained GR-equivalent dynamics into the language of M1's directional momentum. Source correlations deform the Stage, the deformed Stage changes directional momentum comparison, and curvature records the path dependence of that comparison.

3.10 Correspondence with Newtonian Gravity and General Relativity

The field law used in this chapter reaches beyond the inverse-square force: it governs finite geometry, gravitational waves and changing sources. Its correspondence with GR therefore rests on the coupled equations, including the laws for matter and measurement. These determine both the exact agreement and where a different M1 law could change a prediction.

3.10.1 Exact agreement under the same physical laws

The source tensor uses momentum units. In the same frame, the conventional stress–energy tensor is

$$T^{\mu\nu} = cJ^{\mu\nu}, \quad m_0 = \frac{p_f}{c}. \quad (3.65)$$

The second relation identifies the free carrier’s rest-mass parameter. In a zero-shift chart, GR’s lapse is $N = \phi^0_0$ and its spatial metric is $h_{ij} = g_{ij}$. The dictionary expresses the same source and geometry in conventional units and variables.

With this dictionary, the retained field equation is exactly

$$G^{\mu\nu}[g] = \frac{8\pi G_N}{c^4} T^{\mu\nu}. \quad (3.66)$$

On the regular domain, the map between θ and g is invertible and preserves all ten finite field equations. The free-carrier coupling gives the same timelike and null geodesics, and its distribution follows the same collisionless transport law. With the same matter laws, initial and boundary data, and measurement models, this sector gives exactly the same GR solutions and predictions.

The agreement includes the four initial constraints, two local gravitational-wave polarizations and null gravitational characteristics. Under the appropriate regularity conditions, local existence and uniqueness carry over for massive data, and separately for purely massless data supported away from zero momentum. Derivation 3.10A gives these conditions and the comparison premises.

3.10.2 One weak source, three familiar tests

For §3.4’s cold weak source, write $m_{\text{src}} = M_{\text{src}}/c$ and $\ell_g = G_N m_{\text{src}}/c^2$. A slow probe then has

$$\mathbf{a} \simeq -\frac{G_N m_{\text{src}}}{r^2} \hat{\mathbf{r}}. \quad (3.67)$$

This is the Newtonian inverse-square limit. The same source coefficient enters the clock and light comparisons, without separate adjustments of the gravitational coupling.

To turn §3.5’s ideal clock law into an experiment, hold identical clocks at a and b in a static zero-shift geometry, with supports that leave their cycles unchanged. Clock a sends one pulse every m cycles. Let $\mathcal{R}_{a \rightarrow b}$ be the number of received pulses per cycle of clock b . The stationary travel time shifts arrival times but not their separation, so

$$m\mathcal{R}_{a \rightarrow b} = \frac{N_a}{N_b}, \quad m\mathcal{R}_{a \rightarrow b} - 1 \simeq -\ell_g \left(\frac{1}{r_a} - \frac{1}{r_b} \right). \quad (3.68)$$

With $N_a < N_b$, the emitter runs more slowly than the receiver, which records a reduced pulse rate per cycle. The comparison is made through counted events.

For a distant light ray with impact parameter B , the temporal and spatial maps together give

$$\delta_{\text{light}} \simeq \frac{4\ell_g}{B} = \frac{4G_N m_{\text{src}}}{c^2 B}. \quad (3.69)$$

This is the leading GR deflection. Both temporal slowing and spatial deformation contribute to it. The same weak metric gives the propagation delay, while the finite metric and timelike geodesics give orbital precession. All these effects follow from one geometry and one probe coupling.

3.10.3 Finite geometry and horizons

At finite field strength, the supported constant-density solution of §3.6 is the interior Schwarzschild continuum expressed in momentum units. Its pressure, curved volume and temporal weighting enter together.

A complementary vacuum example passes through a horizon without a singular operational map. In ingoing Painlevé–Gullstrand coordinates, let $r_s = 2G_N m_{\text{src}}/c^2$ and $r = |\mathbf{x}| > 0$. The local intervals are

$$d\ell^0 = dx^0, \quad d\ell = d\mathbf{x} + \sqrt{\frac{r_s}{r}} \hat{\mathbf{r}} dx^0. \quad (3.70)$$

This is the Schwarzschild vacuum in a freely falling local frame. Direct substitution satisfies the full field constraints and stationary evolution equations, including at $r = r_s$. Radial null paths obey

$$\frac{dr}{dx^0} = -\sqrt{\frac{r_s}{r}} \pm 1, \quad (3.71)$$

while their speed in the local frame remains c . A clock held at fixed coordinates moves relative to these freely falling frames; its exterior rate is $\sqrt{1 - r_s/r}$, although $\phi^0_0 = 1$ in this shifted map. At the horizon that held worldline ceases to be timelike, while the map itself remains regular.

3.10.4 Complete sources and finite regions

A source must also represent the stresses of its material, including binding and support. For the free-particle source of §3.3,

$$n_i \mathcal{C}^{ij} n_j = \left\langle \sum_a \frac{(n_i p_a^i)^2}{M_a} \right\rangle \geq 0 \quad (3.72)$$

for every spatial direction n_i . A free-particle population therefore has nonnegative normal stress in every direction. Matter with tension needs the stress contributions of its binding fields or internal dynamics as well.

The momentum balance of a finite region includes boundary transport, changes in physical volume and the comparison of local directions across the region. Integrating the local balance retains these flux and geometry terms. Explicit finite-window balances and error bounds quantify these effects. A spatial average alone therefore leaves out information needed for a closed source law.

3.10.5 A supported body with its own clock

A self-bound fluid model supplies a body with its own surface and support. Its pressure vanishes at a finite nonzero density, giving a free surface. Solving the material and gravitational equations together produces a regular spherical body that matches the exterior Schwarzschild geometry without a surface stress layer.

The body's small radial displacements obey an oscillation equation derived from the same material law. In the worked models, an analytic positive bound on every admissible radial eigenvalue establishes linear radial stability. The fundamental mode provides a recurrence, and the changing surface area supplies its clock observable.

A specified weak scalar signal carries that material recurrence outward. The moving body generates sidebands above and below the carrier, each offset by the independently calculated material frequency. Changing the incident carrier frequency leaves those offsets unchanged, while a stationary-body control removes the sidebands. Thus the outgoing signal records the body's cadence rather than a cadence prepared in the input.

For this effective material, support, source, recurrence and the weak outgoing scalar field follow from compatible laws. The calculation connects the body's dynamics to a signal carrying its cadence. Electromagnetic detection and other material clocks require their own dynamical models.

3.10.6 What the dynamical calculations add

The nonlinear evolution in §3.8 advances the same body and its changing geometry for six periods at two specified finite perturbations, with the signal switched off. It measures successive periods and accumulated surface time from the evolved motion. The larger-perturbation extension remains outside the verified range: agreement of local and boundary balances is needed as well as a smooth surface trajectory.

The finite equations for the material and scalar together include opposite momentum exchange, propagated gravitational constraints and transmission across a moving material surface. At finite signal strength, scalar stress outside the body can change its surface mass. The exterior must then evolve along with the body and signal.

Finite material–scalar exchange with gravitational response has been demonstrated in a separate homogeneous calculation. The nonlinear bounded-body evolution, its weak outgoing-signal comparison and the homogeneous finite-exchange solution are distinct results. A full nonlinear body–signal–detector comparison still needs its own coupled solution.

3.10.7 Where M1 could differ

Agreement with GR is exact while the field, matter and measurement laws are shared. A genuine departure must therefore change one of those laws: how internal momentum is redistributed, how its stresses source space, how space evolves, or how a material clock and signal couple to that evolution.

Weak agreement does not uniquely choose the finite laws. Distinct nonlinear field equations can share a linear calibration, and distinct clock couplings can share a free-space frequency. A departure becomes a prediction only after its additional law is specified and its observable consequence derived. §3.6's finite Poisson-feedback model makes one such additional field-law choice; its flat Poisson operator, Euclidean source-volume weights and scalar feedback define a different calculation from the full tensor solution.

M1 requires internal fermic-to-bosic compensation and complete directional conservation. Turning these requirements into a microscopic source and support law, with a consistent common-frame account in curved space, remains the native construction to be completed. The GR results provide its benchmark: a completion must reproduce the shared-law predictions or derive a testable departure from a specified change in the physics.

3.11 What the Gravity Model Establishes

A particle's momentum configuration responds to deformed space and enters its gravitational source. Describing both through the same particle shell connects the motion of matter to the geometry it helps produce.

With the chosen field law, this connection extends to finite geometry and causal, time-dependent dynamics. The four- and six-component forms describe the same system and reproduce GR when the physical laws and comparison conditions are matched. The material examples reach beyond prescribed test particles: one fluid model supplies its own support and radial recurrence, a weak scalar signal carries that recurrence outward, and a separate finite-amplitude calculation follows the body beyond linear motion.

The next task is to give the internally compensated M1 particle an equally complete account. Its momentum redistribution, source and support must follow from its dynamics, and a curved isolated system must satisfy ADMC in a common directional comparison. A native completion then has a clear physical standard: the same momentum that responds to gravity must consistently source it.

4 Quantum Mechanics in Momentum Language

4.1 Quantum momentum and interference

Two preparations can give the same momentum readings and still place their interference fringes in different positions. Take a coherent superposition of two momentum components. Changing their relative phase leaves the probability of either momentum unchanged, but moves the places where the amplitudes reinforce or cancel. A list of momentum probabilities therefore leaves out something physically observable.

The directional shell from Foundations and Gravity remains useful. It tells how a carrier's common core and signed translation appear in opposed directions. Gravity then uses correlations between those components to distinguish, for example, transport along one axis from transport along another. Quantum mechanics adds a further requirement: a description must retain the phases that connect the momentum components of a state.

The object is now a phase-bearing state, Ψ , rather than a trajectory with one definite momentum. Momentum operators act on its amplitudes, and an evolution generator advances their phases. M1 keeps the usual quantum state and probability rules while writing that generator in momentum units. The inherited shell supplies its free magnitude; a first-order spinor realization supplies a local wave equation.

Deformed space changes how that wave propagates and how its phase is compared. The same local and reference distinctions used for particles must therefore survive in the quantum equation. They also matter for a physical clock: a turning phase becomes a clock signal only when an observable internal process repeats.

The reverse gravitational question is just as important. A coherent state that responds to deformation must also enter a consistent account of the momentum that sources it. Its source can depend on interference, transport and internal stresses that no mean shell contains. The chapter develops the propagation and clock examples first, then uses them to make this reciprocal source problem precise.

4.2 Core Terms and Variables

The quantum state carries amplitudes and relative phases. Momentum operators act on that state; their possible readings and probabilities follow from the chosen measurement. The

momentum shell keeps its earlier meaning, while the quantum description adds the information needed for interference.

4.2.1 State and evolution

Symbol	Name and definition	Job
Ψ	Phase-bearing quantum state	Describes a coherent preparation, including its internal components.
ϑ	Relative phase between two amplitudes	Controls how those amplitudes interfere.
$\hat{A}$	An operator acting on the state	Represents a specified observable or generator; the hat distinguishes the operator from a scalar value.
$\widehat{M}_t$	Momentum-unit evolution generator: $(i\hbar/c)\partial_t\Psi = \widehat{M}_t\Psi$ in a fixed-norm representation	Generates phase change with respect to the selected time t .

Complex amplitudes, the Born probability rule and unitary evolution are the quantum premises used here. They are not consequences of the classical shell alone. The time coordinate is written $x^0 = ct$ when it is useful to give all four coordinates length units.

4.2.2 Free core and directional readings

Symbol	Name and definition	Job
p_f	Fermic momentum of the declared free-space realization	Fixes the free massive shell; initially $p_f > 0$.
$\hat{p}_i = -i\hbar\partial_i$	Signed spatial momentum operator in an inertial frame	Generates translations along axis i .
$\widehat{M}_+ = \sqrt{p_f^2 + \hat{\mathbf{p}}^2}$	Positive free core operator	Gives the positive magnitude associated with each free momentum component.
$\hat{p}^{i\sigma} = \widehat{M}_+ + \sigma\hat{p}_i/2$	Opposed directional reading operator, $\sigma = \pm 1$	Combines common core and signed directional imbalance.

Symbol	Name and definition	Job
β, α^i	Constant Hermitian spinor matrices	Supply the first-order realization developed in Section 4.4.

Gravity's slashed index $\ni$ labels the six entries $(1+, 1-, 2+, 2-, 3+, 3-)$. Its indexed L maps still convert between four- and six-component displays. The six reading labels describe momentum projections, not the number of components in Ψ .

The subscript $+$ on $\widehat{M}_+$ marks a positive magnitude. The subscript t on $\widehat{M}_t$ marks a generator's time parameter. A first-order spinor generator has signed spectral branches; it agrees with the positive core on its free positive branch.

4.2.3 Gravity and comparison frames

Symbol	Name and definition	Job
$\phi^a{}_\mu$	Gravity's map: $d\ell^a = \phi^a{}_\mu dx^\mu$	Converts coordinate increments to local orthonormal components.
$N = \phi^0{}_0$	Temporal map coefficient in a Stage-adapted frame	Converts the normal local time increment to the selected coordinate normalization.
$p_{\text{coord},i}, \hat{p}_{\text{coord},i} = -i\hbar\partial_i$	Coordinate momentum and its derivative operator	Distinguishes a coordinate phase gradient from local momentum.
$M, p_{\text{local},a}$	Local core and local spatial momentum	Obey $M^2 = p_f^2 + \mathbf{p}_{\text{local}} ^2$ for the free carrier.
$M_r = NM, p_{f,r} = Np_f, \mathbf{p}_r = N\mathbf{p}_{\text{local}}$	Static, zero-shift reference-expressed momenta	Put the entire local shell into one common reference normalization.
$M_t = -p_{\text{coord},0}$	Scalar chosen-time generator	Gives the future-branch phase rate; it equals M_r in the static zero-shift comparison.

The symbol N abbreviates $\phi^0{}_0$ in calculations. The coordinate label keeps the phase gradient distinct from local and reference-expressed momentum. Spatial variation also changes the volume used to normalize Ψ . The construction in Section 4.5 introduces the corresponding representative ψ when that distinction becomes necessary.

4.3 Phase-bearing states and directional readings

Consider two momentum components in a periodic interval of length $\mathcal{L}$. One has momentum zero; the other has momentum $\hbar k$, where k is a nonzero allowed wave number. At one instant, a scalar Fourier representative with equal amplitudes is

$$\Psi_{\vartheta}(x) = \frac{1 + e^{i\vartheta} e^{ikx}}{\sqrt{2\mathcal{L}}}.$$

The relative phase is ϑ . A momentum measurement returns either value with probability one half, whatever ϑ is. For the position measurement defined by this Fourier representation, however,

$$|\Psi_{\vartheta}(x)|^2 = \frac{1 + \cos(kx + \vartheta)}{\mathcal{L}}. \quad (4.1)$$

Changing the relative phase moves the interference fringes without changing either momentum weight. An incoherent preparation, which chooses one momentum or the other rather than superposing their amplitudes, has the same momentum weights but a uniform position distribution. Interference distinguishes the preparations.

4.3.1 Reading the inherited shell

For a free massive carrier in one inertial frame, each momentum component has the positive core $M(\mathbf{p}) = \sqrt{p_f^2 + |\mathbf{p}|^2}$. Acting on a superposition means applying this same rule to every momentum amplitude:

$$\widehat{M}_+ = \sqrt{p_f^2 + \widehat{\mathbf{p}}^2}, \quad \widehat{p}^{i\sigma} = \widehat{M}_+ + \frac{\sigma}{2} \widehat{p}_i. \quad (4.2)$$

These are the quantum versions of the opposed readings already used in Gravity. Since $M \geq |p_i|$, every reading is at least $M/2$ and hence positive. The pair relations also survive as operator identities:

$$\widehat{p}^{i+} + \widehat{p}^{i-} = 2\widehat{M}_+, \quad \widehat{p}^{i+} - \widehat{p}^{i-} = \widehat{p}_i.$$

All three pairs share the same core. The six entries are six projections of the same momentum organization, not six independently occupied states. Nor does their positivity make them probabilities: they have momentum units and are read through the state and its measurement rule.

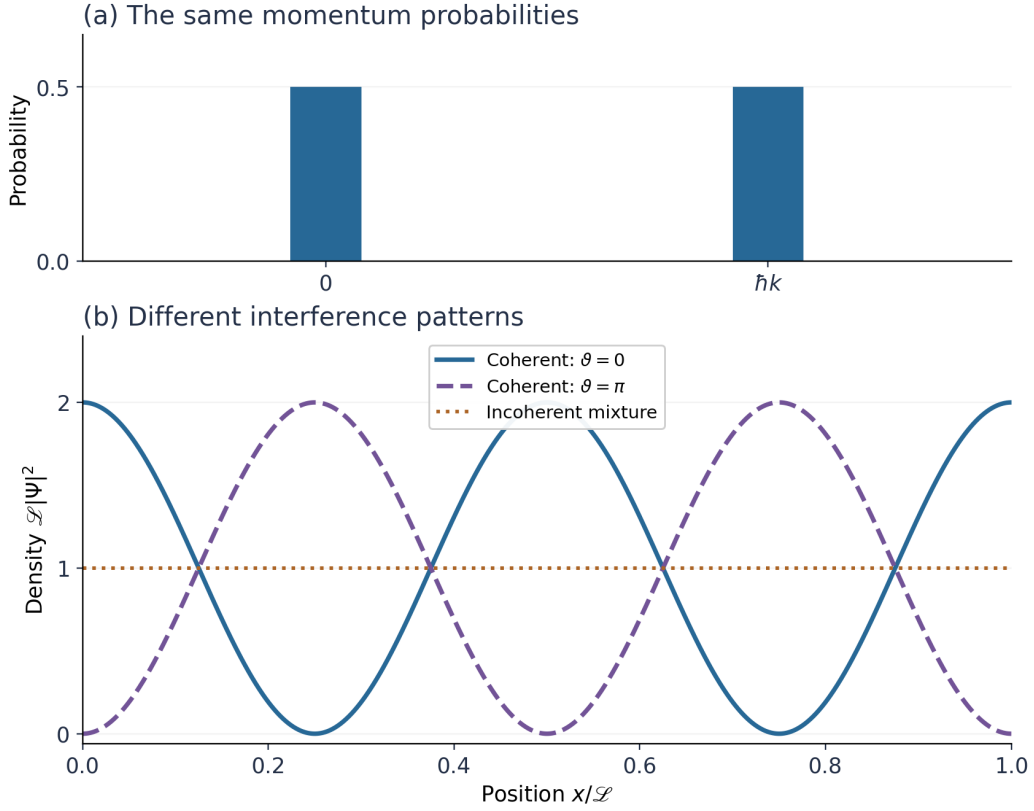


Figure 4.1: Equal momentum weights, different spatial patterns. The upper panel gives the same two momentum probabilities for every preparation shown below. The coherent states with relative phases zero and π have shifted fringes; the incoherent mixture is uniform. The horizontal coordinate is position divided by the periodic length, and the density is multiplied by that length.

In this free representation the reading operators are functions of the same commuting momenta. Consequently, even their complete joint probability distribution is unchanged when ϑ changes in the two-component example. The directional display preserves the momentum information, but it does not restore the relative phase once that phase has been discarded.

4.3.2 Momentum spread and the shell of a mean

A superposition also requires care when taking averages. The shell holds for the operators,

$$\widehat{M}_+^2 - \hat{\mathbf{p}}^2 = p_f^2 I,$$

so its expectation gives

$$\langle \widehat{M}_+^2 \rangle - \langle \hat{\mathbf{p}}^2 \rangle = p_f^2. \quad (4.3)$$

These are averages of squares, not squares of averages. To see the difference, take equal weights at opposite momenta $\pm p_0$ along one axis. Both components have core $M_0 = \sqrt{p_f^2 + p_0^2}$, while their mean signed momentum vanishes. Treating the mean list as a resting carrier would assign it the fermic scale M_0 instead of p_f . The missing information is the momentum spread: $\langle p_x^2 \rangle = p_0^2$.

The same example can be a coherent superposition or an incoherent mixture. Both share these momentum moments. Second moments repair the shell accounting, but they do not by themselves diagnose coherence.

A full directional representation can retain the phase by keeping correlations between *different* momentum arguments, together with the internal state components. Derivation 4.3A gives an invertible construction and the general covariance identity. In the main description, keeping Ψ makes the distinction immediate: momentum readings tell what values can be obtained; the coherent state also tells how their amplitudes interfere.

4.4 The core momentum operator

A momentum component must do more than satisfy the shell: it must also acquire the right phase as the state evolves. In the free positive-core description, the square-root operator supplies both jobs. A component with momentum $\mathbf{p}$ has core $M(\mathbf{p}) = \sqrt{p_f^2 + |\mathbf{p}|^2}$ and evolves with the phase

$$\Psi_{\mathbf{p}}(t) = e^{-icM(\mathbf{p})t/\hbar} \Psi_{\mathbf{p}}(0).$$

Different momentum components generally turn through phase at different rates. Their relative phases then change, moving or reshaping the interference pattern. With the usual quantum phase calibration, the generator law is

$$\boxed{\frac{i\hbar}{c}\partial_t\Psi = \widehat{M}_t\Psi.} \quad (4.4)$$

For free positive-core evolution, $\widehat{M}_t = \widehat{M}_+$. A time-independent self-adjoint generator gives the unitary evolution $\exp(-ic\widehat{M}_t t/\hbar)$, which preserves the state norm. The first derivative in time expresses phase evolution; it does not determine which spatial operator must generate it.

4.4.1 A local first-order realization

The square root is a legitimate operator. In momentum space it simply multiplies each amplitude by its core magnitude. In position space, however, it generally connects amplitudes at separated points rather than acting through a finite list of derivatives at one point. A local first-order equation requires a larger representation.

Dirac's first-order construction (Dirac 1928), expressed in momentum units, keeps the fermic and directed bosic terms visible:

$$\boxed{\widehat{M}_{t,\text{flat}} = \beta p_f + \alpha \cdot \widehat{\mathbf{p}}.} \quad (4.5)$$

Here Ψ is a four-component spinor, and the matrices β and α^i act on its internal components. Their algebra is chosen so that squaring the generator recovers the shell:

$$\beta^2 = I, \quad \{\beta, \alpha^i\} = 0, \quad \{\alpha^i, \alpha^j\} = 2\delta^{ij}I.$$

The braces denote an anticommutator, $\{A, B\} = AB + BA$. In the free case the spatial momentum operators commute, so

$$\begin{aligned} \widehat{M}_{t,\text{flat}}^2 &= p_f^2 I + p_f \{\beta, \alpha^i\} \widehat{p}_i + \frac{1}{2} \{\alpha^i, \alpha^j\} \widehat{p}_i \widehat{p}_j \\ &= (p_f^2 + \widehat{\mathbf{p}}^2) I. \end{aligned} \quad (4.6)$$

The mixed fermic–bosic terms cancel. The remaining spatial terms combine into the squared momentum. This is the explanatory strength of the first-order form: a linear operator on amplitudes recovers the nonlinear scalar magnitude relation when squared.

The matrix choice is an explicit spinor realization of the shell. The six directional readings do not supply these spinor components or derive their statistics. They remain observables that can be expressed in either the four- or six-component momentum display.

4.4.2 Positive core and signed evolution

Squaring a number loses its sign. The same distinction matters here: at fixed momentum the first-order matrix has eigenvalues $+M$ and $-M$, while its magnitude is M on both branches. On the invariant free positive branch it agrees with $\widehat{M}_+$. On the full spinor space it is a signed generator, not a positive core operator.

The enlarged representation makes the equation local. Restricting it to the positive branch requires a momentum-dependent projection, which is generally nonlocal in position space. These are two useful descriptions with different advantages: an explicitly positive free core, and a local first-order spinor equation.

A negative generator eigenvalue does not assign negative physical core to an antimatter particle. The opposite fermic-cycle orientation still carries positive core in M1; relating that orientation to particle and antiparticle field amplitudes requires the field description beyond this one-carrier construction.

Derivation 4.4A gives the spectral projections and the constrained six-component display. The free engine is now explicit: the state retains its amplitudes and relative phases, and a momentum-unit generator advances them while preserving the shell relation in its stated representation.

4.5 Quantum motion in deformed space

A wave packet crossing deformed space samples more than one local momentum comparison. Its phase accumulates against a chosen time standard, its spatial gradients are read through the map ϕ , and its spinor components must be compared between neighboring local frames. The simplest starting point is a static patch in which those conversions are constant.

4.5.1 One shell in local and reference descriptions

Take zero shift and an isotropic spatial map,

$$\phi^0_0 = N > 0, \quad \phi^a_i = b\delta^a_i, \quad d\ell^0 = N c dt, \quad d\ell^a = b dx^a.$$

The scalar $b > 0$ gives the spatial conversion; it is distinct from Gravity's indexed four/six maps L . A coordinate phase gradient $p_{\text{coord},i}$ is read locally as $p_{\text{local},i} = p_{\text{coord},i}/b$. The local core is therefore

$$M = \sqrt{p_f^2 + \frac{|\mathbf{p}_{\text{coord}}|^2}{b^2}}.$$

The static reference expresses the whole shell with the common factor N :

$$p_{f,r} = Np_f, \quad \mathbf{p}_r = \frac{N}{b} \mathbf{p}_{\text{coord}}, \quad M_r = NM = \sqrt{p_{f,r}^2 + |\mathbf{p}_r|^2}. \quad (4.7)$$

A held particle has $M_r = p_{f,r}$, just as in Gravity. During stationary free fall, M_r can remain constant while the local core $M = M_r/N$ changes. The free local shell still contains p_f ; the reference shell contains Np_f . Both describe the same carrier with different comparison standards.

In this constant patch the selected-time spinor generator is

$$\widehat{M}_t = \beta N p_f + \frac{N}{b} \alpha \cdot \widehat{\mathbf{p}_{\text{coord}}}.$$

Its positive branch is $NM = M_r$. Both terms have undergone the conversion required by the same clock and ruler comparison. Multiplying only the fermic term by N while leaving a local spatial derivative unchanged would mix those descriptions.

4.5.2 Spatial variation and probability flow

When N or b varies across the packet, a derivative also acts on the varying conversion factors. The shell magnitude alone no longer specifies the complete wave equation. We use a definite extension: the minimally coupled spinor equation, with neighboring spinors compared by the connection compatible with ϕ and its metric. This supplies a prescribed-background quantum model; its microscopic M1 coupling remains to be derived.

Physical spatial volume is $b^3 d^3x$. Thus the norm of the original local spinor is $\int b^3 \Psi^\dagger \Psi d^3x$. It is convenient to absorb that volume factor into

$$\psi = b^{3/2} \Psi, \quad \int \psi^\dagger \psi d^3x = 1.$$

The state has not changed; the representative now uses ordinary coordinate volume. In this representation the exact static isotropic generator is

$$\boxed{\widehat{M}_t = \beta N p_f + \frac{1}{2} \left\{ \frac{N}{b}, \alpha \cdot \widehat{\mathbf{p}_{\text{coord}}} \right\}}. \quad (4.8)$$

Writing $F = N/b$ makes the derivative term transparent:

$$\frac{1}{2} \{F, \alpha \cdot \widehat{\mathbf{p}_{\text{coord}}}\} = F \alpha \cdot \widehat{\mathbf{p}_{\text{coord}}} - \frac{i\hbar}{2} \alpha \cdot \nabla F.$$

The extra term records the variation of the conversion across the wave. It also makes the probability balance work. The equation for ψ gives

$$\partial_t(\psi^\dagger\psi) + \nabla \cdot (cF\psi^\dagger\alpha\psi) = 0. \quad (4.9)$$

Probability leaving one region enters another. With appropriate decaying, periodic or reflecting boundary conditions, the total norm remains fixed. This current describes probability transport; it is not the gravitational momentum-source tensor.

4.5.3 Beyond the static isotropic patch

A general Stage-adapted map can also tilt the spatial frame relative to the coordinate time lines. Define its spatial part $E^a{}_i = \phi^a{}_i$ and the shift $w^i = (E^{-1})^i{}_a \phi^a{}_0$. Then

$$d\ell^a = E^a{}_i(dx^i + w^i c dt), \quad h_{ij} = \delta_{ab} E^a{}_i E^b{}_j.$$

The inverse spatial metric h^{ij} converts coordinate momentum into its local squared magnitude. For a future ray, the chosen-time generator becomes

$$M_t = N \sqrt{p_f^2 + h^{ij} p_{\text{coord},i} p_{\text{coord},j} - w^i p_{\text{coord},i}}. \quad (4.10)$$

The shift contributes a drift relative to the coordinate grid. The full wave equation also carries the compatible spin transport. For example, in a rotating flat frame the coordinate generator contains both orbital and spin rotation; the shift alone supplies only the orbital part.

If the spatial volume changes with time, the representative $\psi = (\det E)^{1/2} \Psi$ contributes a temporal derivative as well as spatial derivatives. These terms are fixed together by the same spinor equation. Derivation 4.5A constructs them and establishes probability-conserving evolution under its smooth-background and boundary assumptions. The local shell determines the leading momentum balance; the connection, ordering and volume measure determine how a coherent state travels through the varying geometry.

4.6 Phase and physical clocks

A moving wave packet accumulates phase while its peak travels through a spatial pattern. The phase change at a fixed position is therefore different from the phase change followed along the packet. Neither is automatically a clock reading: a clock must turn its internal evolution into an observable recurrence.

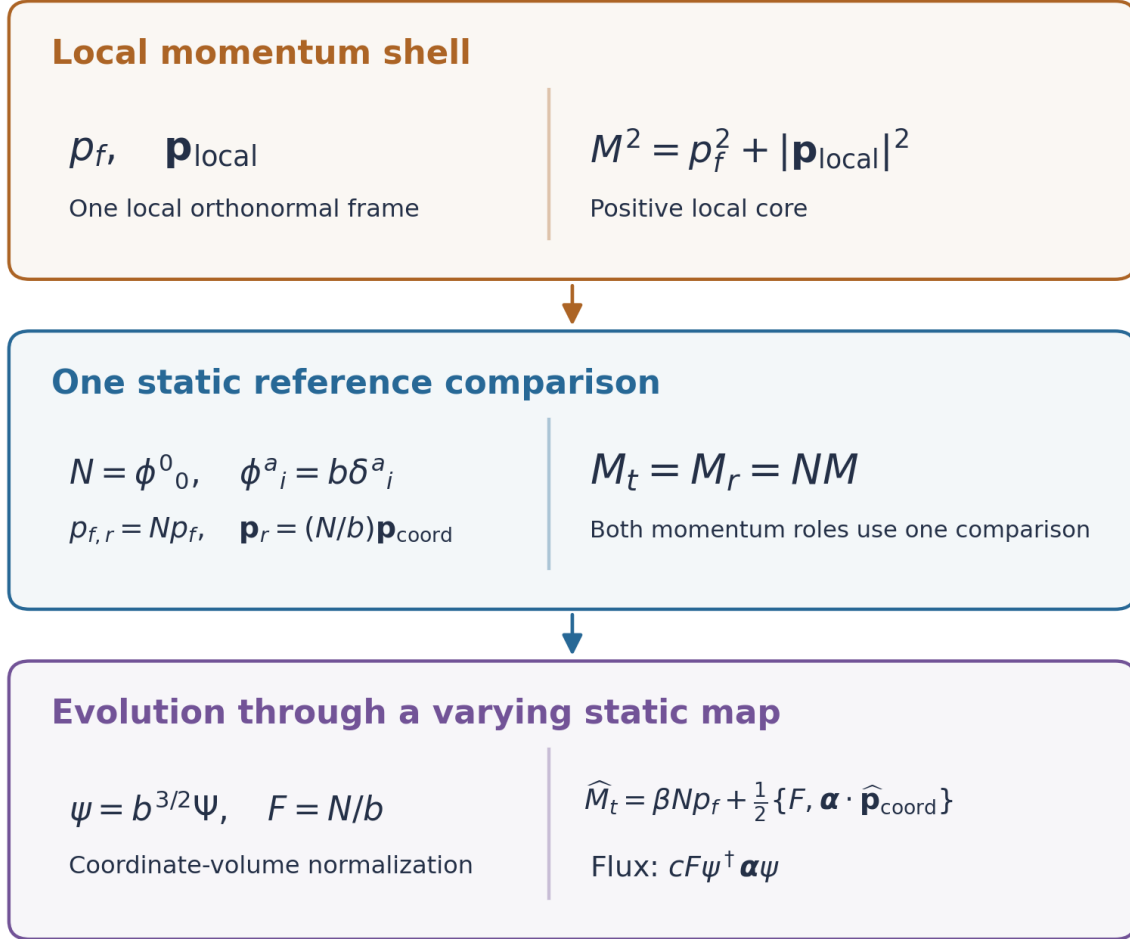


Figure 4.2: The static quantum comparison. The local shell is first expressed through the same temporal and spatial map. The reference generator then advances the state in the selected time. Spatial variation requires ordered derivatives and the corresponding probability flux. The middle panel is the constant-patch relation; the lower panel gives the varying static isotropic realization.

4.6.1 Following phase along a path

For a slowly varying packet write $\Psi \simeq ae^{iS/\hbar}$, where S is the phase expressed in action units. Its spatial gradient gives the coordinate momentum, $p_{\text{coord},i} = \partial_i S$, and its time derivative gives $\partial_t S = -cM_t$. This ray description applies when the amplitude and background vary slowly on the local wavelength scale, away from turning points and branch crossings.

Use the future generator from Equation 4.10. The packet velocity is $\dot{x}^i = c \partial M_t / \partial p_{\text{coord},i}$. Following the phase along that motion gives

$$\begin{aligned} \frac{dS}{dt} &= \partial_t S + p_{\text{coord},i} \dot{x}^i \\ &= -cM_t + p_{\text{coord},i} \dot{x}^i \\ &= -\frac{cNp_f^2}{M}. \end{aligned} \tag{4.11}$$

The transport term subtracts the phase variation traversed by the packet. The shift cancels in the result. Along the same massive ray, $d\tau/dt = Np_f/M$, so $dS/dt = -cp_f d\tau/dt$.

In the static reference notation this becomes

$$-\frac{1}{\hbar} \frac{dS}{dt} = \frac{c}{\hbar} \frac{p_{f,r}^2}{M_r}.$$

The magnitude agrees with Gravity's ideal fermic-cycle rate. By contrast, the phase rate at a fixed coordinate is $cM_t/\hbar$. For a null ray the leading phase is constant *along* the ray even though a detector at a fixed location can register an oscillating wave. Phase propagation and an internal fermic cycle are different physical comparisons.

4.6.2 A clock with an observable signal

A common phase multiplying an isolated state cancels from every expectation value. A clock signal instead needs a relative phase that changes a measured quantity. A two-state internal system supplies a simple example.

Call the two distinguishable configurations $|L\rangle$ and $|R\rangle$. Let their local momentum-unit generator be

$$\widehat{M}_{\text{int}} = M_0 I + g\sigma_x, \quad M_0 > g > 0,$$

where σ_x interchanges the configurations and g is the coherent coupling in momentum units. Its two stationary values are $M_0 \pm g$. Prepare the system in $|L\rangle$ and hold it at rest in a zero-shift

comparison, with N effectively uniform across its size. Assume that its internal dynamics couples to proper time without being altered by the support. Then

$$\tau(t) = \int_0^t N(t') dt', \quad |\Psi_{\text{int}}(t)\rangle = e^{-icM_0\tau/\hbar} \left[\cos\left(\frac{cg\tau}{\hbar}\right) |L\rangle - i \sin\left(\frac{cg\tau}{\hbar}\right) |R\rangle \right].$$

The probability of finding the system in $|R\rangle$ is

$$\boxed{P_R(t) = \sin^2\left(\frac{cg\tau(t)}{\hbar}\right)}. \quad (4.12)$$

The common M_0 phase has disappeared. The gap $2g$ produces the oscillation. Identically prepared trials measured after different waiting times reveal this signal without assuming that repeated measurements leave one running clock undisturbed.

At constant N , the proper period is $\pi\hbar/(cg)$ and the reference period is $\pi\hbar/(cNg)$. A smaller N therefore gives a slower reference clock. The effect follows from the specified internal coupling, not from declaring every phase factor a clock.

4.6.3 What a common slowdown requires

One working clock does not establish that all material processes rescale together. A system can have several transition gaps, coherent couplings and relaxation rates. To behave as the same system running more slowly, all observable internal dynamics must share the time conversion.

Compare the baseline internal generator $\widehat{M}_{\text{int},0}$ with its reference-time counterpart $\widehat{M}_{\text{int},r}$. For a finite-dimensional closed sector with all states and observables accessible, every process shares one time rescaling exactly when

$$\widehat{M}_{\text{int},r} = \Lambda \widehat{M}_{\text{int},0} + aI. \quad (4.13)$$

Here Λ is the common rate factor and aI changes only an unobservable common phase. Scaling one transition probability is weaker: it need not scale the coherent phase changes that drive other measurements. For an open system, relaxation and noise must be included in the comparison of the full dynamics.

Derivation 4.6A proves the criterion and separates the ray-phase identity from the internal-clock premise. The two-state example shows how a coherent quantum process can realize the gravitational clock comparison. Extending that result to material clocks requires their internal interactions and support conditions to produce the same rescaling.

4.7 Correspondence and controlled limits

The momentum-unit generator gives a common starting point for three familiar descriptions: relativistic spinor motion, a slow quantum envelope and an approximately classical trajectory. Each uses the same state dynamics at a different level of resolution. A matched barrier experiment then tests whether a gravitational comparison has changed the physical experiment or only its normalization.

4.7.1 Dirac dynamics in standard units

Multiplication by c converts the chosen-time generator into the conventional energy-unit Hamiltonian:

$$\widehat{H} = c\widehat{M}_t.$$

For the flat first-order realization, using $p_f = m_0 c$ gives

$$\widehat{H} = \beta m_0 c^2 + c\alpha \cdot \widehat{\mathbf{p}}, \quad i\hbar\partial_t\Psi = \widehat{H}\Psi.$$

This is the free Dirac equation in Hamiltonian form. The correspondence is exact: the amplitudes, phase evolution and measured probabilities are unchanged. Core momentum remains the primary quantity in the M1 description; energy units provide the familiar comparison.

4.7.2 The slow envelope

At low directed momentum, most of the free phase rate comes from the fermic scale. The positive magnitude expands as

$$M = p_f + \frac{p^2}{2p_f} - \frac{p^4}{8p_f^3} + O\left(\frac{p^6}{p_f^5}\right).$$

The first term produces a rapid common rest phase. The slower change of the wave envelope is governed by the excess above it. In the standard Dirac block representation, write

$$\Psi = e^{-icp_ft/\hbar} \begin{pmatrix} \varphi \\ \xi \end{pmatrix},$$

where φ and ξ are two-component amplitudes. The three Pauli matrices σ act on these components. In the low-momentum positive branch, ξ is small. Its leading equation gives

$$\xi \simeq \frac{\sigma \cdot \hat{\mathbf{p}}}{2p_f} \varphi.$$

Substituting into the equation for φ multiplies two first-order momentum operators. Since the free momentum derivatives commute, $(\sigma \cdot \hat{\mathbf{p}})^2 = \hat{\mathbf{p}}^2$, and

$$\boxed{\frac{i\hbar}{c} \partial_t \varphi = \frac{\hat{\mathbf{p}}^2}{2p_f} \varphi.} \quad (4.14)$$

In energy units this is $i\hbar \partial_t \varphi = -\hbar^2 \nabla^2 \varphi / (2m_0)$. The familiar second-order Schrödinger operator appears because the small spinor component has been eliminated, not because the underlying relativistic equation has changed its spatial order.

A weak, smooth static deformation adds the expected leading potential. Write $N = \phi^0_0 = 1 + \nu$ and take ν and $b - 1$ to be of the same small order as p^2/p_f^2 , with spatial gradients controlled on the envelope scale. Removing the constant free rest phase then gives

$$\frac{i\hbar}{c} \partial_t \varphi \simeq \left[p_f \nu + \frac{\hat{\mathbf{p}}_{\text{coord}}^2}{2p_f} \right] \varphi.$$

For the Newtonian comparison, $\Phi = c^2 \nu$ turns the first term into $m_0 \Phi$ after multiplication by c . Away from this weak, slowly varying regime, the ordered spinor equation must be used before reducing the envelope. In a constant isotropic patch, for example, the kinetic excess in reference units is $N |\mathbf{p}_{\text{coord}}|^2 / (2p_f b^2)$: the same temporal conversion multiplies the whole local excess.

Derivation 4.7A supplies the elimination, gradient ordering and approximation conditions.

4.7.3 From waves to trajectories

The ray form used for the path-phase calculation also recovers particle motion. When $\Psi \simeq a e^{iS/\hbar}$, derivatives acting on the rapidly varying phase dominate those acting on the amplitude. The spinor equation then requires

$$g^{\mu\nu} \partial_\mu S \partial_\nu S = -p_f^2, \quad g = \phi^\top \eta \phi. \quad (4.15)$$

Its future branch is the chosen-time generator already obtained from the momentum map. The rays follow

$$\dot{x}^i = c \frac{\partial M_t}{\partial p_{\text{coord},i}}, \quad \dot{p}_{\text{coord},i} = -c \frac{\partial M_t}{\partial x^i}.$$

Thus the same wave equation returns Gravity's free-carrier trajectories. The compatible connection transports amplitude and spin at the next order. Near a turning point, where a propagating wave becomes an evanescent one, the simple ray approximation must give way to wave matching.

4.7.4 Matched gravitational barriers

Tunnelling is sensitive to small changes in the forbidden momentum because that momentum enters an exponential. It is therefore also sensitive to comparing different incident states or different apparatus standards by mistake.

Begin with a constant N and use proper distance ℓ across a one-dimensional barrier. Let $u(\ell)$ be its local additive momentum coefficient and C the stationary reference generator value. The local incident value is $\epsilon = C/N$. Dividing the consistently converted stationary equation by N gives

$$[-i\hbar\alpha^1\partial_\ell + \beta p_f + u(\ell)I]\psi = \epsilon\psi.$$

The temporal map coefficient has disappeared. Two experiments with the same local incident value, local barrier height, proper width, fermic scale and apparatus therefore have the same transmission. The incoming and outgoing probability fluxes must also be normalized in that same comparison. Derivation 4.7A checks the statement by exact spinor matching, without relying on a tunnelling exponent alone.

A spatially varying N is a different experiment. If the distant incident value C is held fixed, the local value $C/N(\ell)$ changes along the barrier. In a forbidden interval the local decay momentum satisfies

$$\kappa_{\text{local}}^2 = p_f^2 - \left[\frac{C}{N(\ell)} - u(\ell) \right]^2.$$

Where the variation is slow enough for a semiclassical approximation, the amplitude contains

$$\exp\left[-\frac{1}{\hbar} \int \kappa_{\text{local}} d\ell\right].$$

Nonuniform deformation can consequently change the transmitted fraction substantially. The appendix verifies such a change with the full wave equation and conserved flux. It is the gravitational sensitivity of the specified, consistently coupled spinor model, shared by its metric description. An additional quantum-gravitational effect would require an additional physical interaction, compared with the same preparation and apparatus—not a conversion applied to only one term of the shell.

4.8 Quantum sources and reciprocal gravity

A coherent carrier both responds to deformed space and contributes momentum to its source. The propagation equation describes the first direction. The second requires a local account of core, transport and stress that preserves the state's interference and the momentum exchanged with other systems.

Gravity already supplied a simple reason to keep correlations. Equal counterstreams along x and equal counterstreams along y can have the same total core and zero net spatial momentum. Their six mean directional readings then agree, but their transport differs. Products of readings on each carrier retain that difference; products of the aggregate means lose it.

4.8.1 Correlate before averaging

For the free positive-core quantum description, the corresponding kinetic operator is

$$\hat{j}_{\text{kin}}^{\ni j} = \hat{p}^{\ni} \widehat{M}_+^{-1} \hat{p}^j. \quad (4.16)$$

The inverse core weights the two directional readings on the same carrier. In an incoherent kinetic ensemble, summing these products with weights per physical volume returns Gravity's source $J^{\ni j}$. It is a momentum-weighted correlation, not an ordinary covariance.

The shell fixes its contraction before averaging:

$$-\eta_{\ni j} \langle \hat{j}_{\text{kin}}^{\ni j} \rangle = p_f^2 \langle \widehat{M}_+^{-1} \rangle. \quad (4.17)$$

The average inverse core appears here, rather than the inverse of the average core. Each momentum component brings its own weight. This is another place where replacing a distribution by a mean shell changes the source.

The reference conversion must preserve that weight too. For a static comparison at one common $N = \phi_0^0$,

$$\frac{p^{\ni} p^j}{M} = \frac{p_r^{\ni} p_r^j}{N M_r}.$$

The numerator contains two reference factors, while the core denominator contains one. The remaining factor of N restores the original local source. Conversion of physical volume is a separate step.

4.8.2 Interference enters the local source

The kinetic expression still leaves out a distinction exposed by the two-component state. Its momentum weights stay fixed when the relative phase changes, but its spatial interference pattern moves. A local coherent source can depend on the cross terms between those amplitudes as well as on their separate weights.

To calculate that source, the model must say how the coherent amplitudes couple to deformation. A specified matter action is one way to give propagation and source together: varying the matter state gives its equation of motion, while varying the deformation gives the momentum source. The action here is the mathematical rule for their joint dynamics. A free shell or a table of diagonal readings does not select that rule.

A scalar example makes the additional content concrete. Take a normalized free complex scalar state Ψ with positive-core evolution. To carry the inverse-core weighting into its quadratic source, form $\zeta = \widehat{M}_+^{-1/2}\Psi$. This re-encodes the same state, retaining its relative phases. The minimally coupled quadratic scalar action specified in Derivation 4.8A acts on ζ .

Write this scalar model's source as $J_0^{\mu\nu}$. In its flat free realization, the local components are $u = J_0^{00}$, $j_i = J_0^{0i}$ and $s_{ij} = J_0^{ij}$: common core density, core flux and momentum transport. They satisfy $|\mathbf{j}| \leq u$, so the opposed densities are

$$\boxed{d^{i\pm} = u \pm \frac{1}{2}j_i \geq \frac{u}{2} \geq 0.} \quad (4.18)$$

All pairs share the same local core. The scalar equation also gives their directional balance,

$$\partial_0 d^{i\pm} + \partial_j \left(j_j \pm \frac{1}{2}s_{ji} \right) = 0, \quad x^0 = ct.$$

For the normalized scalar state, with periodic or decaying boundaries, the integrated densities equal the expectation values of the free directional reading operators. Spatial phase is retained, and the transport terms carry changes between neighboring regions.

Those transport terms include gradients produced by interference. They can contain a negative transverse stress even while the common core and all opposed densities remain positive. A sum of nonnegative classical terms p_i^2/M cannot reproduce that stress. Thus a coherent source can satisfy positive directional balance without being a classical collection of independently moving carriers at every point.

4.8.3 Probability, total core and local source

The spinor model used for propagation has a different source. For its standard minimally coupled symmetric source, interference between two positive-branch modes can produce a negative local core-density patch. The probability density remains positive, and the spatially integrated core remains positive. Derivation 4.8A gives an explicit two-mode state exhibiting all three properties.

There is no contradiction between this example and the scalar construction. They use different matter models and different source expressions. Together they show why a positive probability, a positive total core and a positive local source are separate requirements. Writing the source in six components preserves its information; it cannot change a negative common-core value into six positive opposed densities.

Even a successful free scalar source is not uniquely fixed by its kinetic limit. Additional coherent gradient terms can leave the free diagonal readings and integrated totals unchanged while altering the local stress. Their physical selection must come from the interaction and the way its source is measured.

4.8.4 Complete reciprocal accounting

An interaction transfers momentum between participants and can entangle their internal states. The complete source must include that interaction and the systems that support or drive it. Counting only the responding carrier omits part of the exchange; counting the same interaction in both carriers' cores counts it twice.

A finite two-carrier example in Derivation 4.8A demonstrates coherent entanglement with equal and opposite momentum transfer and positive conserved global directional charges. It is a constructive accounting example, not a derived local gravitational interaction. A native continuum description must supply the local source, its reciprocal response and the deformation dynamics within one common balance.

The quantity associated with internal recurrence also must not be mistaken for the entire gravitational source. Gravity's source blocks give

$$-\eta_{\mathfrak{J}} J^{\mathfrak{J}} = \mathcal{M} - \text{tr } \mathcal{C},$$

whereas the weak temporal deformation responds to $\mathcal{M} + \text{tr } \mathcal{C}$. A null kinetic carrier has zero fermic-recurrence contraction but a nonzero temporal source. Momentum does not need a fermic clock to gravitate.

A complete coherent interaction must determine both the response to deformation and its source, with material and Stage contributions included in the same directional balance.

4.9 A changing Stage and the one-particle boundary

A spatially uniform change of the Stage can alter phase evolution without changing a carrier's spatial momentum label. Translation symmetry preserves that label. The changing Stage alters how the same directed content enters the generator.

The homogeneous case used by Expansion leaves the fermic scale fixed and supplies a positive bosic translation yield $\chi(t)$. Its first-order generator is

$$\boxed{\widehat{M}_\chi = \beta p_f + \chi(t)\alpha \cdot \hat{\mathbf{p}}.} \quad (4.19)$$

Here $\mathbf{p}$ is the conserved spatial label used in Chapter 5. In the coordinate notation of Section 4.5 it is $\mathbf{p}_{\text{coord}}$. The same operator follows from the homogeneous map $\phi^0_0 = 1$, $\phi^a_i = \chi^{-1}\delta^a_i$, with zero shift and the volume-rescaled state. The connection and changing-volume terms combine as derived in Derivation 4.5A.

At fixed time, the positive branch magnitude is

$$M_\chi = \sqrt{p_f^2 + \chi^2 p^2}.$$

For a narrow packet that follows this branch, its translation along axis k is

$$\boxed{\dot{x}_k = c \frac{\partial M_\chi}{\partial p_k} = \chi^2 c \frac{p_k}{M_\chi}.} \quad (4.20)$$

The two factors of χ follow from differentiating the dressed squared momentum. The conserved label $\mathbf{p}$ continues to represent the same spatial momentum content. Its local expression $\mathbf{p}_{\text{local}} = \chi\mathbf{p}$, phase rate and coordinate translation change with the Stage. The history $\chi(t)$ is supplied here; deriving it requires the Stage's dynamics and its complete source.

4.9.1 Following an instantaneous branch

A unitary equation preserves the full state norm, but a changing generator need not preserve its instantaneous positive branch. For a massive mode, changing χ changes the relative weight of the fermic and bosic matrices. It therefore changes their eigenvectors as well as their eigenvalues.

If the change is slow compared with the separation of the spectral branches, and transitions remain small over the interval, a packet can approximately follow one branch. A rapid change can instead mix the instantaneous branches while conserving the full norm. Derivation 4.5A provides both a sudden-change example and the conditions for a controlled slow-history

approximation. Particle creation cannot be inferred from this mixing alone; that interpretation requires a field state and a specified particle measurement.

4.9.2 Beyond a fixed carrier

A one-carrier state retains momentum and internal coherence within a fixed particle description. A field state can also contain coherence between different particle-number sectors, including the vacuum and a particle pair. That information is absent from a one-carrier density operator.

The distinction limits what can be inferred from positive one-carrier source examples. If a source operator were positive in every field state and had exactly zero vacuum expectation, it would annihilate the vacuum. It could then have no matrix element connecting the vacuum to a pair. A source with such pair coherence cannot satisfy all of those conditions at once. The vacuum source mean, its fluctuations and a detector's response must therefore be distinguished rather than identified with one positive density.

Real unbound momentum can still source gravity without forming a stable particle. Waves and propagating spatial deformation already make that possibility part of the physical picture. A fluctuation variance, however, does not by itself establish additional positive mean core. A field description must determine the source and its measurement together.

For the supplied homogeneous history, the packet law above gives a concrete next step. Chapter 5 follows massive and lightlike propagation and compares emitted patterns with material standards at a later epoch. A quantum law for the Stage and a complete field-source theory remain separate tasks.

4.10 What the quantum chapter establishes

A coherent momentum state carries more than a list of possible readings. Its amplitudes and relative phases determine interference, while its momentum operators preserve the directional shell and its second-moment balance. The six-component display makes common core and opposed imbalance visible without replacing the state that connects them.

Core momentum also organizes quantum evolution. The positive free magnitude and the local first-order spinor generator give complementary representations, with agreement on the free positive branch. Consistent conversion through ϕ extends the specified spinor model to deformed space, including the derivative ordering, spin transport and volume factors needed for probability conservation. Its slow envelope and rays recover the corresponding Schrödinger and gravitational particle limits; matched barriers distinguish physical spatial variation from a change of normalization.

A coherent internal process can turn that evolution into a clock signal. Its observable recurrence depends on relative phase and a specified internal coupling. A reciprocal gravitational

description asks more: the same interaction must determine the response and the complete coherent source. The scalar and spinor examples make that requirement sharper, but do not yet select the microscopic M1 interaction or a quantum law for the Stage.

The next chapter takes the homogeneous Stage history as supplied. It asks how conserved spatial momentum translates and how signals are compared when the basic yield changes. The quantum generator now provides the link from that changing yield to phase and motion.

5 Space Expansion and Stage Dressing

5.1 Space Expansion and the Conservation Problem

Cosmic expansion confronts M1 with an immediate tension. Momentum conservation is the framework's founding principle. Yet the expansion of space appears to change the momentum expressed by freely travelling light. How can momentum remain conserved when the universe itself seems to produce a non-conservative result?

The question becomes especially sharp across cosmological distances. A photon may travel for billions of years before reaching us. When it arrives, its wavelength is longer, and the event that produced it appears to unfold more slowly. Something differs between emission and observation. But what has changed?

M1 separates two physical roles. Momentum is the actor of change; space is the stage on which that change becomes motion and distance. If a distant photon arrives stretched, there are therefore two places to look. The actor may have changed, or the stage may have changed how the actor's momentum is expressed.

Suppose first that the actor changes. A photon might gradually surrender momentum to space itself. Nothing in logic forbids an exchange, but the word *loss* hides the hard part. Momentum is directed. It cannot simply fade away: something must receive its direction, transmit it onward, or balance it with an equal and opposite change. A drain into space would therefore require a physical receiver and a closed conservation ledger. Without them, the proposed loss only moves the problem elsewhere.

The expansion engine developed here takes the other route. In a homogeneous stage, spatial translation symmetry protects the carrier's spatial momentum content. The directed momentum does not disappear, and the intrinsic scale $p_f = m_0 c$ remains fixed. What changes is how the stage turns those quantities into translation.

A simplified physical picture begins with an elongation of the particle's internal basic cycle. The carrier retains the same basic momentum content, but that momentum is expressed through a longer cycle, shifting its frequency.

Picture a violin string resonating with a note. Now imagine the violinist slowly sliding her finger toward the scroll, lengthening the part of the string that is free to resonate. The pitch falls. In the same way, elongating the internal basic cycle shifts its frequency without taking momentum from the carrier.

The directional asymmetry still belongs to the cycle as a whole. But as the internal cycle elongates, the same asymmetry completes fewer cycles in a given interval. Its *translation yield* in real space is therefore reduced: the carrier has not lost directed momentum, but the altered cycle produces less translation from it.

The quantum chapter prepared the mathematical form of this distinction. Its first-order core-momentum operator contains a fermic slot and a directed bosic slot. Expansion leaves the fermic slot fixed while dressing the bosic one. The carrier can therefore retain the same spatial momentum label even as the generator frequency and the motion produced from that label change with cosmic epoch.

The analogy gives the physical picture; the operator supplies the law. The second power of the stage factor in the massive translation equation is neither inferred from the string nor added by hand. It follows when the stage-dressed generator is differentiated with respect to directed momentum.

Light provides the cleanest first signal of the construction, but not its only test. One engine must preserve the lightlike propagation law, recover the correct history of freely moving massive bodies, connect wavelength redshift with duration stretching, and remain compatible with stable local rods, clocks, atoms, and bound orbits. It must also say clearly what it has not yet supplied.

This chapter builds that engine. It establishes homogeneous stage-dressed kinematics and several scoped local realizations. It does not derive the history of the stage, a cosmological field equation, or a complete observational model. Those are later burdens. The task here is narrower: to show how a changing stage can alter the expression of conserved momentum across epochs without turning momentum conservation into a metaphor.

5.2 Core Terms and Variables

The expansion engine adds one stage variable and two coordinate descriptions to momentum and operator notation already established. This section fixes their meanings and governing relations in one place. The mechanism and derivation begin in the next section.

5.2.1 Inherited momentum and operator terms

Fermic momentum p_f is the fixed intrinsic scale,

$$p_f = m_0 c.$$

The **directed bosic momentum** is $\mathbf{p}$, with magnitude $p = |\mathbf{p}|$. For an oriented unit direction $\hat{k}$, its signed component is

$$p_k = \mathbf{p} \cdot \hat{k}.$$

The **inertial core-momentum magnitude** is

$$M = \sqrt{p_f^2 + p^2}.$$

The **canonical momentum operator** is $\hat{\mathbf{p}} = -i\hbar\nabla$. The **flat core-momentum operator** is

$$\hat{M} = \beta p_f + \alpha \cdot \hat{\mathbf{p}}.$$

The constant c retains its inherited role as the intrinsic local propagation scale.

5.2.2 Stage state and dressed generator

The dimensionless **homogeneous stage state** is $\chi(t) > 0$, with endpoint values

$$\chi_e = \chi(t_e), \quad \chi_o = \chi(t_o).$$

The observation epoch is the default reference, so $\chi_o = 1$ unless another normalization is stated. When the normalization remains explicit, the **normalized stage ratio** is

$$X(t) = \frac{\chi(t)}{\chi_o}, \quad X_o = 1.$$

Compact operator formulas use χ in the normalized convention; otherwise X takes its place.

The **stage-dressed core-momentum operator** and its positive instantaneous magnitude are

$$\boxed{\hat{M}_\chi = \beta p_f + \chi(t) \alpha \cdot \hat{\mathbf{p}}}, \quad \boxed{M_\chi = \sqrt{p_f^2 + \chi^2 p^2}}.$$

Here M is the inertial magnitude, while M_χ is the instantaneous eigenvalue or principal-symbol magnitude of the dressed generator. The spatial momentum content $\mathbf{p}$ remains a separate quantity.

Translation yield names the stage-conditioned relation between directed momentum and generated motion. Its governing law is

$$\dot{x}_k = \chi^2 c \frac{p_k}{M_\chi}.$$

Derivation 5.2A supplies the operator algebra and group-velocity theorem.

5.2.3 True-frame and bound-frame variables

The common homogeneous history parameter is t . The **true-frame coordinate** x_T records propagation on the homogeneous chart. The **bound-frame coordinate** x_B reports lengths using co-scaling local rods:

$$x_B = \frac{x_T}{X}.$$

The associated clean-branch scale factor and fractional history rate are

$$a_B = \frac{1}{X} = \frac{\chi_o}{\chi}, \quad H = \frac{\dot{a}_B}{a_B} = -\frac{\dot{X}}{X}.$$

The clean branch uses matched local clocks, so t also serves as the local time variable in the comparisons below. The function $H(t)$ is supplied rather than derived in this chapter.

5.2.4 Propagation and signal labels

For lightlike propagation, the **history kernel** is

$$\mathcal{D}_\chi(t_e, t_o) = c \int_{t_e}^{t_o} \chi(t) dt.$$

It labels a path interval, not a complete observational distance.

For neighboring signal features, δt_e and δt_o denote endpoint separations in the common parameter t . The symbols Δt_e and Δt_o denote durations reported by the emitter's and observer's local clocks.

Observed wavelength redshift is

$$1 + z = \frac{\lambda_o}{\lambda_e}.$$

On the matched-standard branch developed below, the conditional endpoint relation is

$$1 + z = \frac{\chi_e}{\chi_o}.$$

Momentum labels conserved spatial content, M_χ labels the dressed generator value, X compares stage states, and bound-frame variables describe measurements made with local standards.

5.3 Stage Dressing and Massive Motion

Expansion acts on the part of the core-momentum generator that carries translation. The fermic scale $p_f = m_0 c$ continues to sustain the carrier's intrinsic cycle, while the directed bosic term is expressed through the changing stage:

$$\hat{M}_\chi = \beta p_f + \chi(t) \alpha \cdot \hat{\mathbf{p}}.$$

Because $\chi(t)$ is homogeneous, it changes no spatial direction and introduces no spatial gradient. The carrier therefore keeps its momentum label $\mathbf{p}$. What changes is the magnitude generated from that content. Squaring the operator gives

$$M_\chi = \sqrt{p_f^2 + \chi^2 p^2}.$$

The bosic contribution now enters through χp . As the internal bosic cycle elongates, the same directed content turns through that cycle at a different rate, and the generator magnitude changes with it.

5.3.1 From the dressed generator to motion

For a narrow positive-branch packet, motion along the selected direction k is determined by how the energy-unit generator cM_χ changes with the signed momentum component p_k :

$$\begin{aligned} \dot{x}_k &= \frac{\partial(cM_\chi)}{\partial p_k} \\ &= c \frac{\partial}{\partial p_k} \sqrt{p_f^2 + \chi^2 p^2} \\ &= \chi^2 c \frac{p_k}{M_\chi}. \end{aligned}$$

Thus

$$\dot{x}_k = \chi^2 c \frac{p_k}{M_\chi}.$$

The two powers of χ belong to one operation. Stage dressing places χ on the bosic term, so that term contributes $\chi^2 p^2$ to the core-momentum magnitude. Differentiating that magnitude with respect to directed momentum brings the full factor into translation. The dressed generator itself therefore yields the χ^2 law.

The law immediately retains the expected directional behavior. At $\chi = 1$, it returns to inertial motion, $\dot{x}_k = cp_k/M$. If $p_k = 0$, the carrier has no motion along $\hat{k}$. Reversing the chosen orientation reverses both p_k and $\dot{x}_k$, while M_χ remains unchanged.

For light, $p_f = 0$ and $M_\chi = \chi|p|$. One power then cancels from the ratio, leaving forward propagation at

$$\dot{x}_T = \chi c.$$

Massive and lightlike motion are therefore two limits of the same stage-dressed generator.

5.3.2 Why the bosic slot carries the dressing

The two operator slots have different physical jobs. The fermic term carries the intrinsic scale tied to the carrier's identity. The bosic term carries the directional asymmetry through which that carrier translates. Expansion changes the resonating length through which the bosic cycle is expressed, so its factor belongs on the bosic term.

This placement also joins the chapter's two kinematic requirements. Keeping p_f fixed preserves the intrinsic fermic scale. Dressing the bosic term by one power of χ gives the lightlike true-frame speed χc . The first-order two-slot construction is therefore

$$\hat{M}_\chi = \beta p_f + \chi \alpha \cdot \hat{\mathbf{p}}.$$

The massive law then follows from the same operator rather than from a separate rule for material particles.

5.3.3 Free massive motion in bound-frame units

The observation epoch now sets the normalization $\chi_o = 1$, so $X = \chi$ and

$$a_B = \frac{1}{\chi}.$$

A local observer uses rods that co-scale with X . After the common recession of the bound-frame coordinate grid is removed, the carrier's peculiar velocity is the local-motion term

$$v_B = a_B \dot{x}_T.$$

Substituting the stage-dressed translation law gives

$$\begin{aligned} v_B &= a_B \chi^2 c \frac{p}{\sqrt{p_f^2 + \chi^2 p^2}} \\ &= c \frac{p/a_B}{\sqrt{p_f^2 + (p/a_B)^2}}. \end{aligned}$$

Hence

$$\boxed{v_B = c \frac{p/a_B}{\sqrt{p_f^2 + (p/a_B)^2}}}.$$

The conserved content p has not decayed. Local rods express it through the epoch-dependent quantity p/a_B , so the measured velocity changes while the underlying spatial momentum label remains fixed.

This is exactly the special-relativistic free-particle velocity in a spatially flat FLRW description when

$$\boxed{p \equiv q},$$

where q is conserved comoving momentum. The correspondence identifies two descriptions of the same homogeneous free-carrier history: M1 keeps the spatial momentum content p , while the bound frame reports its changing expression p/a_B .

Differentiating the velocity gives

$$\dot{v}_B = -Hv_B \left(1 - \frac{v_B^2}{c^2}\right).$$

Slow massive motion therefore falls as a_B^{-1} , while light remains at the local speed c .

5.3.4 A massive-motion placement diagnostic

Massive motion also tests where the stage factor belongs. If one factor of χ dresses the complete inertial magnitude instead of the basic slot, the true-frame law is $\dot{x}_T = \chi c p / M$. Bound-frame conversion then cancels the stage dependence:

$$v_B^{\text{whole}} = a_B \chi c \frac{p}{M} = c \frac{p}{M}, \quad \dot{v}_B^{\text{whole}} = 0.$$

Both placements retain the lightlike speed, but only basic-slot dressing produces the massive FLRW drag law. Together with the fixed fermic scale, this selects the basic-slot form within the first-order two-slot multiplicative architecture.

The expansion engine therefore reaches the exact free-carrier history for every speed once the stage history is supplied. Derivation 5.2A carries the operator algebra and scoped placement result, and Derivation 5.3A gives the full bound-frame conversion and drag relation.

5.4 Light Propagation and Endpoint Comparison

Light follows the same dressed generator as a massive carrier. Setting $p_f = 0$ gives

$$M_\chi = \chi |p|.$$

Substituting this magnitude into the translation law leaves

$$\dot{x}_T = \chi(t) c.$$

One power of χ remains after the dressed magnitude cancels the other. The basic-slot construction therefore carries massive and lightlike motion in one law: fermic content changes the massive response, while a purely basic carrier translates at the stage-conditioned rate χc .

5.4.1 A fixed spatial pattern with a changing phase rate

Write the lightlike spatial momentum as

$$\mathbf{p} = \hbar \mathbf{k}.$$

The stage state depends on time but not position. It therefore changes no spatial phase gradient, and the external wavevector remains fixed during flight:

$$\dot{\mathbf{k}} = 0.$$

The generator frequency changes with the stage. For a lightlike carrier,

$$\omega_\chi(t) = \frac{cM_\chi}{\hbar} = \chi(t)c|\mathbf{k}|.$$

The signal consequently carries two distinct features. Its external spatial pattern is set by the conserved wavevector, with true-frame coordinate wavelength

$$\lambda_T = \frac{2\pi}{|\mathbf{k}|},$$

while its phase turns at the changing rate $\omega_\chi(t)$. The pattern remains fixed on the homogeneous true-frame chart as the basic cycle changes its temporal expression. A wavelength redshift appears when that transported pattern is read against local rods at the two endpoints.

5.4.2 The path accumulated through the stage

A signal emitted at t_e and observed at t_o samples every stage state between them. Integrating its translation rate gives the true-frame path interval

$$\mathcal{D}_\chi(t_e, t_o) = c \int_{t_e}^{t_o} \chi(t) dt.$$

The integral measures the translation accumulated by light through the supplied history. Different histories with the same endpoint ratio can therefore produce different path intervals even though they produce the same matched-standard redshift.

The clean bound frame uses $a_B = 1/\chi$ in the normalized convention. Its conformal time satisfies

$$\boxed{d\eta = \frac{dt}{a_B} = \chi(t) dt}, \quad \mathcal{D}_\chi = c \int d\eta.$$

If the observation normalization remains explicit, the exact relation is $d\eta = Xdt$. The stage-dressed path kernel and the usual conformal propagation kernel are therefore the same path written in the two coordinate descriptions. Their conformal causal reach is identical: once the history is supplied, stage-dressed kinematics does not enlarge the light cone or solve the horizon problem.

Observable distance and brightness relations require the later transport layer built on this interval.

5.4.3 Neighboring features on the same path

Now consider two neighboring features emitted by the same source and received by the same observer. Over their short separation, take the source and observer to retain the same true-frame coordinate separation. The two features then traverse equal paths:

$$c \int_{t_e}^{t_o} \chi(t) dt = c \int_{t_e + \delta t_e}^{t_o + \delta t_o} \chi(t) dt.$$

Subtracting the common part and keeping the first-order endpoint contributions gives

$$\boxed{\chi_o \delta t_o = \chi_e \delta t_e}.$$

The later feature must wait a different interval at observation because translation proceeds under a different stage state there. The propagation law therefore carries the endpoint ratio directly into the separation between neighboring features:

$$\frac{\delta t_o}{\delta t_e} = \frac{\chi_e}{\chi_o}.$$

These intervals belong to the common history parameter. Local rods and clocks now turn them into reported wavelengths and durations.

5.4.4 Wavelength at the endpoints

On the clean branch, a local true-frame rod scales with $X(t) = \chi(t)/\chi_o$. The transported wavelength λ_T is fixed, while its numerical reading in local rod units scales as $1/X$. Comparing the same spatial pattern at emission and observation gives

$$1 + z \equiv \frac{\lambda_o}{\lambda_e} = \frac{\chi_e}{\chi_o}.$$

The signal arrives redshifted because the observer reads its conserved external pattern with a different local rod. Momentum has not drained from the carrier along the path; the endpoint standards express the same transported pattern differently.

5.4.5 Duration at the endpoints

Matched local clocks retain the same rate against the common history parameter. Their reported durations therefore inherit the neighboring-feature relation:

$$\frac{\Delta t_o}{\Delta t_e} = \frac{\delta t_o}{\delta t_e} = \frac{\chi_e}{\chi_o}.$$

Combining this with the wavelength ratio gives

$$\Delta t_o = (1 + z)\Delta t_e.$$

Wavelength redshift and duration stretching are thus two readings of one endpoint comparison. The stage history determines the path, equal paths determine the neighboring-feature separation, and co-scaling rods with matched clocks determine what emitter and observer report. Derivation 5.4A gives the full endpoint expansion and regime assumptions.

5.5 Bound Frames and Local Standards

A universal stage state cannot be interpreted by applying the free-propagation law indiscriminately to every atom, laboratory, planet, or galaxy. A bound system has an equilibrium set by its kinetic and interaction terms. The local question is therefore not whether cosmological motion is simply switched off, but whether the whole bound configuration and the standards used to measure it transform coherently.

5.5.1 The clean bound-frame construction

Normalize the stage state by

$$X = \frac{\chi}{\chi_o}.$$

The clean construction takes a local true-frame rod to scale linearly with X ,

$$\ell_T(X) = \ell_o X,$$

while local tick frequencies remain fixed relative to the common history parameter. A true-frame interval is then reported in local rod units through

$$\boxed{x_B = \frac{x_T}{X}}, \quad \boxed{a_B = \frac{1}{X}}.$$

A bound separation that co-scales as $R_T = R_o X$ is locally constant:

$$R_B = \frac{R_T}{X} = R_o.$$

A fixed true-frame separation between freely separated systems behaves differently. In bound-frame units it is multiplied by a_B , and therefore appears to expand as X falls. The same transform can thus describe local stability and cosmological separation without declaring bound systems exempt from the stage.

5.5.2 Local light and radar measurements

True-frame lightlike motion is

$$\dot{x}_T = cX.$$

During one fixed local tick, the true-frame light path scales as X . The rod used to measure it scales by the same factor. Their ratio is therefore constant, so a local observer continues to report the intrinsic speed c .

Radar ranging gives the same result over a sufficiently short round trip. If $H\Delta t_T \ll 1$, the stage ratio is effectively constant during the measurement. For a local radius that co-scales as $r_T = r_o X$, the round-trip light time is, to leading adiabatic order,

$$\Delta t_T = \frac{2r_T}{cX} = \frac{2r_o}{c}.$$

A fixed local clock counts the same number of ticks. Co-scaling rods, light paths, and bound separations can therefore remain stable in local measurement even while their true-frame lengths vary with the stage.

5.5.3 Adiabatic scope

For a bound system with characteristic internal time τ_{dyn} , the natural regime is

$$H\tau_{\text{dyn}} \ll 1.$$

The stage then changes little during one internal cycle, allowing the system to follow a sequence of near-equilibrium states. This condition does not force every interaction to co-scale correctly. It identifies the regime in which the proposed local transform can be tested without violent nonequilibrium driving.

The clean transform is therefore a target rather than a universal proof. Electromagnetic binding must produce the required rod and spectral behavior. Gravity must preserve bound orbital scales and timing. Different clock families must not acquire forbidden relative drifts, and the remaining interaction sectors must be compatible with the same standards.

The next section tests that target in a hydrogenic electromagnetic proxy and a Newtonian bound orbit, then checks linear matter growth at a supplied background history. These controlled cases make the construction concrete without closing every interaction sector.

5.6 Scoped Realizations and Correspondence Checks

The clean bound frame sets a definite physical target. Local rods must scale as X , local clock frequencies must remain fixed, and a bound system must keep the same size and timing when measured by those standards. The low-speed basic generator supplies two powers of X in the kinetic response. The question is then which interaction scaling lets an electromagnetic atom or a Newtonian orbit meet the target.

In both proxies, the answer is one power of X . That exponent is not inserted merely because it works downstream; it is selected independently by the required bound scale in each sector.

5.6.1 A hydrogenic electromagnetic proxy

After the rest phase is removed, write the normalized dressed magnitude as $M_X = \sqrt{p_f^2 + X^2 p^2}$, with $p_f = m_e c$ for the electron. Its low-speed expansion is

$$c(M_X - p_f) \simeq \frac{X^2 \mathbf{p}^2}{2m_e}.$$

Let the Coulomb strength scale provisionally as

$$e^2(X) = e_o^2 X^n,$$

with $\hbar$ and m_e fixed. For a hydrogenic state, $p \sim \hbar/r$, so balancing the X^2 kinetic contribution against the Coulomb attraction gives

$$a_0(X) \propto \frac{X^2 \hbar^2}{m_e e^2(X)} \propto X^{2-n}.$$

The clean-branch rod target is $a_0(X) = X a_{0,o}$. It therefore selects

$$\boxed{n = 1, \quad e^2(X) = e_o^2 X}.$$

The same exponent fixes the spectral scale. Hydrogenic transition energies vary as

$$\Delta E(X) \propto \frac{e^4(X)}{X^2} \propto X^{2n-2},$$

so $n = 1$ gives

$$\boxed{a_0(X) = X a_{0,o}, \quad \Delta E(X) = \Delta E_o}.$$

Atomic lengths co-scale with local rods while electronic transition frequencies remain fixed. The dimensionless coupling read against the local light scale is likewise constant:

$$\alpha_X = \frac{e^2(X)}{\hbar c X} = \frac{e_o^2}{\hbar c}.$$

The electromagnetic proxy therefore supplies the rod and clock behavior used in the endpoint comparison. Its first-power law is selected inside the hydrogenic adiabatic model; fuller electromagnetic and material standards require their own realization.

5.6.2 A Newtonian bound orbit

The gravitational proxy begins from the same X^2 low-speed kinetic response. Write the effective Newtonian coupling provisionally as

$$G_{\text{eff}}(X) = G_N X^g.$$

For an adiabatic circular orbit, the angular action $J = pr$ is conserved. Balancing the kinetic and attractive terms gives the orbital scale

$$r_{\text{orbit}}(X) \propto \frac{X^2 J^2}{m^2 G_{\text{eff}}(X) M_s} \propto X^{2-g}.$$

The clean branch requires the orbit to co-scale with local rods, $r_{\text{orbit}} \propto X$. This independently selects

$$\boxed{g = 1, \quad G_{\text{eff}}(X) = G_N X}.$$

The orbital radius and true-frame velocity then obey

$$\boxed{r_{\text{orbit}}(X) = X r_{\text{orbit},o}}, \quad v_T \propto X.$$

Their common factor cancels from the period:

$$T_T \sim \frac{r_{\text{orbit}}}{v_T} \propto X^0.$$

The Newtonian orbit therefore retains its size against local rods and its timing against the matched clock. As in the atomic case, the clean target selects the needed first power within the proxy rather than deriving a complete microscopic sector law.

5.6.3 Linear matter growth at a fixed history

The same powers reappear in the linear growth equation. In the pressureless, linear, quasi-static regime, take $H(t)$ as supplied and use the selected Newtonian scaling $G_{\text{eff}} = G_N X$. The source acquires two powers from the kinetic response and one from the coupling:

$$X^3 = \underbrace{X^2}_{\text{kinetic response}} \underbrace{X}_{\text{Newtonian coupling}}.$$

That product is exactly the homogeneous volume conversion between true-frame and bound-frame matter density,

$$\bar{\rho}_B = X^3 \bar{\rho}_T.$$

The density contrast therefore obeys

$$\ddot{\delta} + 2H\dot{\delta} = 4\pi G_N \bar{\rho}_B \delta.$$

This is the standard linear matter-growth equation at the same supplied $H(t)$ and $\bar{\rho}_B(t)$. The matching friction and source terms are consequences of the same kinetic and coupling powers that stabilize the local proxies.

5.6.4 What these checks establish

The clean-branch requirement selects first-power scaling independently in the hydrogenic and Newtonian models. With those powers, atomic lengths, electronic clock frequencies, orbital radii, orbital periods, and linear matter growth fit one common local-standard construction. Derivation 5.6A provides the detailed balances, growth derivation, and regime ledger.

The result is a coherent electromagnetic/Newtonian realization, not yet an all-sector law. Its value is sharper than an assumed fit: two different bound systems select the same coupling exponent, and that exponent then completes the X^3 structure required by linear growth.

5.7 What the Expansion Engine Establishes

The expansion problem began with a conservation seam. Distant signals arrive stretched, yet directed momentum cannot simply vanish into an unnamed reservoir. Botic-slot stage dressing resolves the homogeneous free-carrier kinematic seam by separating conserved spatial content from the generator that turns that content into phase evolution and motion. It does not yet close the full conservation ledger of stage and carrier together.

5.7.1 The established engine

In a homogeneous background, spatial translation symmetry preserves the carrier label $\mathbf{p}$. Expansion leaves the intrinsic fermic scale $p_f = m_0 c$ fixed and dresses the directed bosic slot of the core-momentum generator. For the resulting magnitude $M_\chi = \sqrt{p_f^2 + \chi^2 p^2}$, group velocity gives the chapter's anchor law:

$$\dot{x}_k = \chi^2 c \frac{p_k}{M_\chi}.$$

Given the bosic-slot ansatz, no second multiplier is attached downstream: the second power follows when the dressed generator is differentiated with respect to conserved momentum. The law recovers inertial motion at $\chi = 1$, respects directional reversal and zero projected momentum, and reduces to true-frame lightlike motion at speed χc .

On the clean bound-frame branch, the same law reproduces the exact special-relativistic FLRW velocity and drag relations when M1's conserved spatial content is identified with comoving momentum. Massive motion also distinguishes the placement: whole-generator dressing would leave peculiar velocity constant, while bosic-slot dressing produces the required drag.

Light follows the same engine. Its external wavevector remains fixed while its generator frequency changes with χ . The signal accumulates the standard conformal propagation interval, and neighboring features inherit the corresponding endpoint timing ratio. The causal reach is unchanged for a supplied history.

5.7.2 Conditional local realization

Turning propagation into measurements requires a local-standard dictionary. On the clean branch, true-frame rods co-scale with $X = \chi/\chi_o$, bound-frame lengths are reported through $x_B = x_T/X$, and matched clocks retain a fixed rate against the common history parameter. These premises give the endpoint relations $1 + z = \chi_e/\chi_o$ and $\Delta t_o = (1 + z)\Delta t_e$.

The same local target selects the interaction powers in two controlled bound-system checks. The hydrogenic proxy requires $e^2(X) = e_o^2 X$ to produce co-scaling atomic lengths and fixed electronic transition frequencies. The Newtonian orbit independently requires $G_{\text{eff}}(X) = G_N X$ to produce co-scaling orbital radii and fixed local timing. With that Newtonian power, the X^2 kinetic response and the X coupling combine into the X^3 density conversion that yields standard linear pressureless growth.

These are coherent realizations in their stated regimes. Full electromagnetic, material, relativistic-gravity, strong, and weak closure remains to be built.

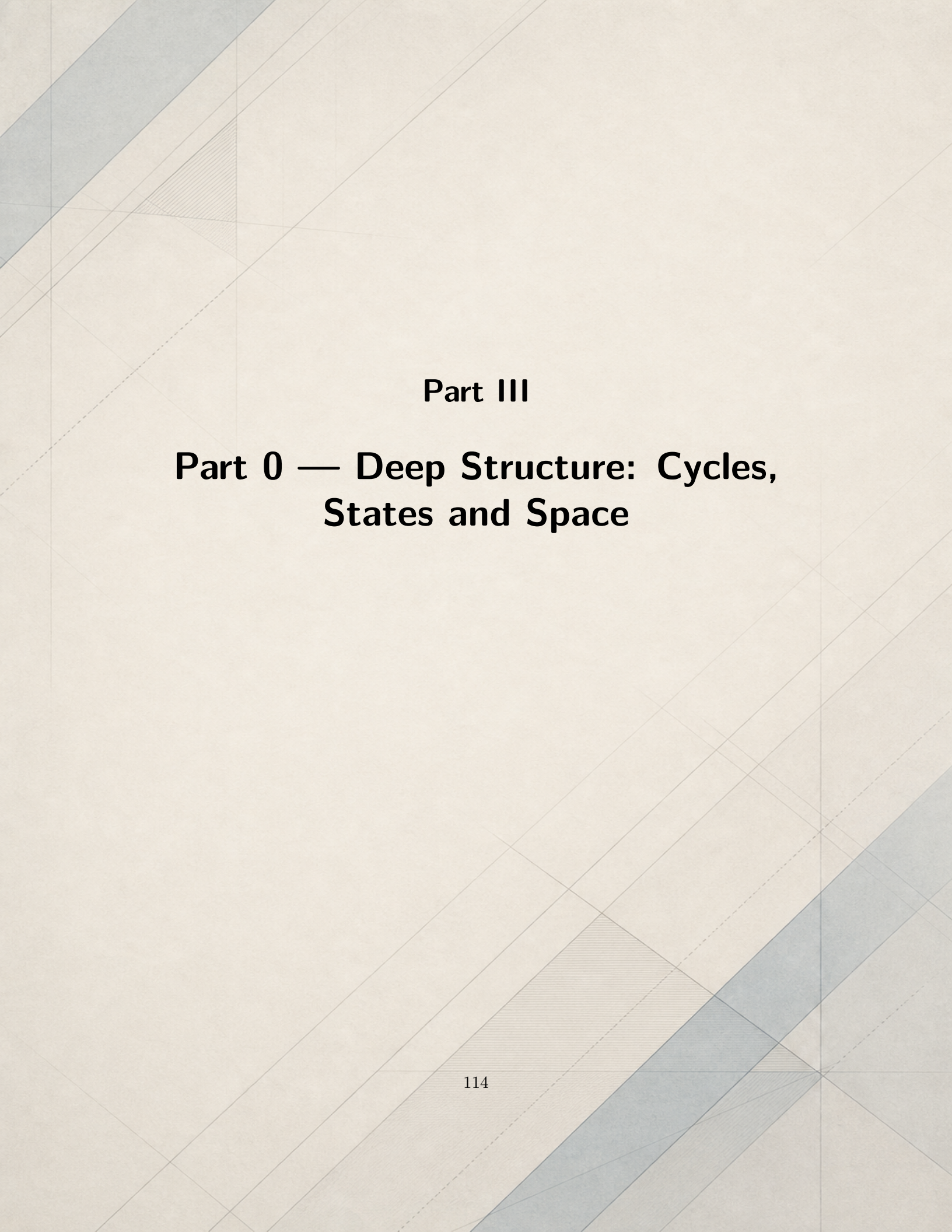
5.7.3 Open dynamics and handoff

Once $H(t)$ is supplied, the homogeneous scale history $a_B(t)$, free-carrier motion, light paths, endpoint ratios, and—under the scoped Newtonian realization—linear matter growth are fixed. The load-bearing cosmological problem is therefore the law that determines $H(t)$ itself.

This chapter derives no stage action or evolution equation for $\chi(t)$, and it does not settle the gravitational source weight of the dressed generator. Conserved spatial content, instantaneous generator value, gravitational source weight, and any future stage-plus-carrier constraint remain distinct ledger questions; ADMC is unchanged here.

Radiation-era perturbations, nonlinear growth, lensing, complete signal transport, thermal history, and observational fits belong to the later cosmological layer. The deeper theory must first produce a stage history; Tier 3 can then turn that history and completed sector laws into distances, brightness, growth, lensing, degeneracy tests, and falsifiers.

The Tier 2 gain is concrete: one generator now governs massive and lightlike motion while preserving homogeneous spatial momentum content, its placement passes the massive-motion diagnostic, and its clean local branch selects a common first-power scaling in two distinct bound-system proxies. The remaining center of gravity has moved from kinematics to dynamics—from how a given history is expressed to why that history occurs.



Part III

Part 0 — Deep Structure: Cycles, States and Space

6 A First Picture of M1 Geometry

6.1 From Momentum to Geometry

A particle can cross a room while its internal processes repeat. A clock carried with it counts those repetitions; its trajectory records the translation. Both belong to one physical system. In momentum-first language, the fermic role sustains internal cycling, while the bosic role carries translational expression. Geometry offers a way to picture how those roles can coexist, how they can change, and how a recurring organization can persist as it moves.

The earlier chapters described momentum configurations and the laws governing their motion. Here we ask what underlying organization could support them. What closes within a particle when an internal cycle completes? What changes when the particle translates? How might a longer internal cycle alter its momentum and its clock rate? These questions lead from the geometry of a particle's internal organization toward its relation with the surrounding space.

Part 0 approaches that deeper structure through three increasingly demanding pictures. This chapter begins with an ordinary torus: two cycles drawn on a familiar ring-shaped surface. It connects the fermic and bosic roles, follows translation through a helix, and illustrates dilation, resonant loops and waves. The next chapter, *Beyond the Torus*, introduces richer phase and orientation information through Clifford geometry and the shell's Clifford representation. The final chapter, *M1, Strings and the Structure of Space*, examines what string and matrix descriptions can contribute to these questions. Their connection to M1 will depend on matching physical quantities and laws, not merely on finding similar shapes.

The ordinary torus gives us a deliberately simplified starting point. Its cycles carry useful relationships, but its familiar shape supplies more visual detail than the physics has established. We will use it as a map of selected internal changes, not as a literal particle surface in the room. A separate trace will show the particle's translation through that room. Keeping those two uses of geometry visible lets a simple drawing do substantial work without asking every distance in it to be a physical length.

6.2 Two Cycles on a Torus

Imagine bending a tube into a ring and joining its ends. There are now two ways to go around. One follows the ring around its central opening; the other circles the tube itself. A point on

the surface can advance in either direction, or in both at once. To specify its position, we need to say how far it has progressed around each cycle.

Progress around a cycle is its *phase*. A full turn advances the phase by 2π , returning that coordinate to its starting value. In this picture, the major, ring-following phase is u , and the minor, tube-following phase is v . We use the major cycle to picture the basic momentum role and the minor cycle to picture the fermic role. A moving marker then displays the changing pair of phases.

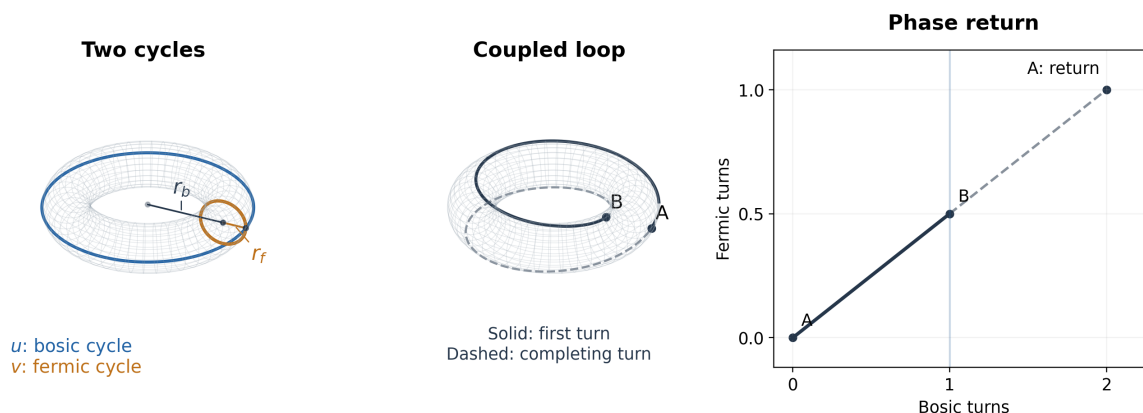


Figure 6.1: Geometry, coupled motion and phase return. The first panel separates the major cycle, in blue, from the minor cycle, in amber; r_b measures the major radius to the tube centre and r_f the tube radius. The middle panel follows a loop through both cycles: its solid half reaches B, and its dashed half completes the return to A. The phase chart counts two basic turns for one fermic turn. The torus is an internal phase display with proportions chosen for legibility.

The surface path in Figure 6.1 couples the two advances. Follow it from A. After one major turn, the marker has reached the opposite minor phase, B: it has come around the central opening but has not returned around the tube. After a second major turn, it completes the minor turn as well. The pair of phases returns to A.

This is a selected two-to-one resonance. *Resonance* here means that the two advances remain locked in a ratio that permits a combined return. Other ratios give other paths. A rational ratio closes after suitable whole numbers of turns; an irrational ratio does not return exactly under steady two-phase motion. The torus accommodates all of these possibilities. A physical model must determine which of them can persist.

The two-to-one example is useful because the incomplete first return is easy to see. Returning both phase coordinates tells us where the marker is; describing its orientation requires further information. For now, the closed curve records a simpler fact—two periodic coordinates can complete a shared cycle even when they complete different numbers of turns.

The drawing also gives each cycle a size. Write r_b for the major display radius and r_f for the minor display radius. Changing either alters the shape we see. Connecting those sizes to momentum requires an additional physical rule; the phase labels alone do not supply one. We can already use the torus to follow recurrence before making that further identification.

6.3 Momentum and a Moving Clock

The two cycles organize the momentum of one carrier. Fermic momentum p_f supplies its stable internal scale, basic momentum p its translational component, and core momentum M their combined magnitude. In the free inertial setting,

$$M^2 = p_f^2 + p^2.$$

The triangle in Figure 6.2 makes this composition visible. At the front-centre point Q , p_f and M share an origin, and the blue p arrow completes the triangle. Measure θ from the fermic direction to M : the fermic fraction is $p_f/M = \cos \theta$, while the basic fraction is $p/M = \sin \theta$. The illustrated momentum ratio is chosen so that M follows the loop's local tangent.

In the constant-speed manifold picture, propagation through the internal geometry maintains speed c . The triangle resolves its direction into a fermic component $c \cos \theta$ and a basic component $c \sin \theta$. Increasing basic momentum at fixed p_f tilts the resultant away from the fermic direction. The underlying propagation speed stays fixed; its fermic component decreases.

A clock counts fermic cycles. Use reference time t in a chosen inertial frame. Let f_0 be the carrier's cycling frequency when at rest in that frame, and f its frequency when moving through it. Foundations gives the moving frequency as

$$f = f_0 \frac{p_f}{M} = f_0 \cos \theta.$$

The moving carrier retains its fermic scale but completes fewer cycles during the same reference interval. At rest, $p = 0$, so $M = p_f$, $\theta = 0$ and $f = f_0$. As basic momentum increases, M grows and the frequency falls. The changing angle pictures the slowing of a physical clock.

The same momentum balance governs translation. Choose an oriented direction $\hat{k}$ in ordinary space. The signed component $p_k = \vec{p} \cdot \hat{k}$ records the basic momentum along it, and

$$\dot{x}_k = c \frac{p_k}{M}.$$

The dot denotes change per unit reference time. Along the forward direction of motion, $p_k = p$, so the translation speed is $v = c \sin \theta$. The basic fraction sets how quickly the carrier advances; the fermic fraction sets how quickly it cycles.

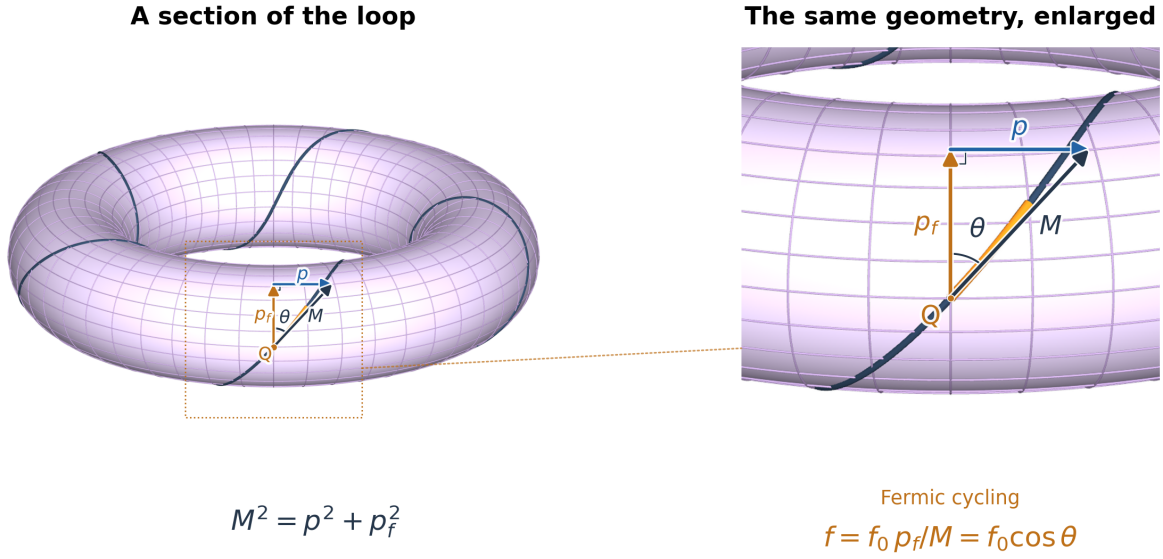


Figure 6.2: Momentum composition and fermic cycling. The close-up enlarges the boxed region. This selected loop makes five fermic turns per basic turn; the amber segment highlights the path near Q . There, M follows the local tangent, p_f shares its origin, and p completes the triangle. The angle θ directly shows the fermic fraction p_f/M . The frequency relation uses Foundations' inertial clock law, with f_0 the resting frequency and f the moving frequency. The torus displays the momentum directions; its drawn path length does not set the clock rate.

The moving clock records a time τ , calibrated so that each fermic cycle counts as $1/f_0$. Counting the same cycles gives $f dt = f_0 d\tau$, hence

$$d\tau = dt \frac{p_f}{M}.$$

Since $v/c = p/M$, the momentum relation also gives $p_f/M = \sqrt{1 - v^2/c^2} = 1/\gamma$, where γ is the Lorentz factor. This is the inertial time-dilation relation expressed through momentum and cycling.

The earlier two-to-one example showed when the two phases return together; the fermic frequency tells how quickly the clock cycles. To picture translation next, we follow the distance advanced during each bosic turn.

6.4 Translation and Pitch

Translation depends on the wave's advance in the bosic direction and on how that advance is directed along the carrier's motion. Write v_b for the wave velocity in the bosic direction. In the constant-speed manifold picture,

$$v_b = c \sin \theta = c \frac{p}{M}.$$

Keep the torus in view while the carrier translates. Place each turn of the bosic phase at the position the carrier has reached. Phase winds around the display axis while displacement advances along it, producing a helix. The torus's symmetry axis is aligned with $\hat{k}$, and its centre follows that straight axis. The helix records the bosic phase alone: its radius stays fixed rather than moving inward and outward with the fermic phase.

Define the *pitch angle* α as the angle between the helix tangent and the circumferential direction in the plane perpendicular to $\hat{k}$. The angle compares axial advance with advance around the bosic cycle. Use a calibrated helix picture in which those two distances share the same length scale. For steady motion along the forward branch,

$$\dot{x}_k = v_b \tan \alpha.$$

Here v_b is the circumferential component, not the total speed along the drawn helix. A larger v_b gives faster translation at the same pitch angle; a larger α gives faster translation at the same v_b . The angles have different jobs: θ resolves momentum into its fermic and bosic parts, while α relates bosic advance to translation.

At $\alpha = 45^\circ$, axial and circumferential advances are equal. The translation law becomes

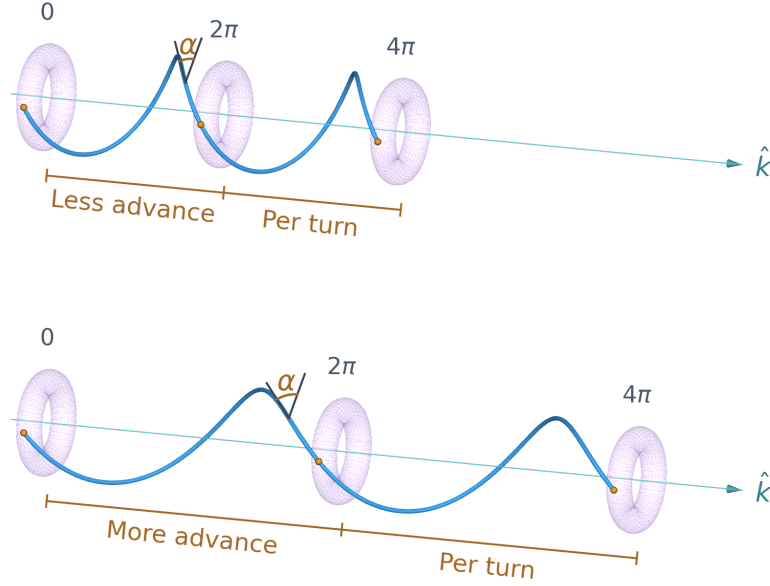


Figure 6.3: Two basic phase histories with different pitch angles. The panels use the same basic wave velocity, cycle size, camera and axial scale, isolating the change in α . Each angle is drawn between the local circumferential direction and the helix tangent. Small transparent tori mark centre positions at basic phases 0 , 2π and 4π ; the rulers compare axial advance per turn. The helix radius equals $r_b + r_f$, the sum of the torus's major and minor display radii. The carrier centre follows the straight axis, while the helix records its basic phase history.

$$\dot{x}_k = v_b = c \sin \theta = c \frac{p}{M},$$

recovering the forward free-inertial relation. This 45° case belongs to the calibrated helix picture, rather than to an angle measured directly on the perspective drawing. For free inertial motion, v_b and α must together satisfy the momentum law. Whether this equal-advance relation is fixed, or the basic asymmetry can yield a smaller pitch angle in other circumstances, remains an open question.

Bosonic dilation changes the internal cycle's extent and can also change its propagation and translation response. Cycle size alone therefore does not determine how quickly the carrier moves. The next section follows those changes together.

In the earlier two-turn example, the selected pair of phases returns after two basic turns, but at a new position. Internal closure and return to the same place are different events. The distance advanced per turn is also distinct from a spatial wavelength, which compares equal wave phases at one instant. Identifying the two requires a relation between internal cycling and the spatial wave.

6.5 Two Ways to Dilate the Geometry

A cycle can grow in extent without changing how many times a pattern winds around it. Imagine spreading the same repeated pattern along a longer closed path. Each repetition then occupies more length. If the number of wave repetitions stays fixed and momentum scales inversely with their length, the corresponding momentum scale falls.

This supplies a useful conditional interpretation of the torus radii. Increasing the minor scale can depict a reduced fermic scale; increasing the major scale can depict a reduced basic expression. The inverse-length rule is the extra assumption that connects the drawing to momentum. Without it, enlarging a torus is simply changing a picture.

6.5.1 Fermionic dilation and gravity

M1 interprets gravitational response through dilation of the internal fermionic geometry and the associated reduction of realized fermic momentum. The minor-cycle change in Figure 6.4 gives that interpretation a visible form. It must be read in the same comparison as the momentum change, rather than as a measured enlargement of a particle surface.

In the static, zero-shift comparison of the Gravity chapter, p_f denotes the particle's fermic momentum in the free-space realization chosen as the reference. The temporal coefficient ϕ^0_0 of the deformation map expresses the gravitational change in that common reference:

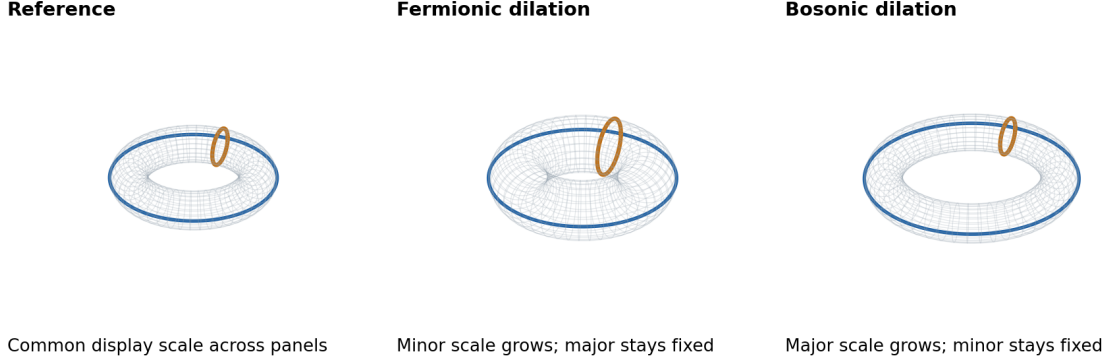


Figure 6.4: Two distinct illustrative dilations. The reference torus is compared with a larger minor, fermionic scale and a larger major, bosonic scale. Each comparison changes one display radius at a time. With a fixed number of repetitions and an inverse-length momentum rule, the enlarged cycle carries a lower corresponding momentum scale. The panels use a common display scale, not a calibrated physical size; they distinguish the two geometric changes rather than specifying a full gravitational trajectory or a cosmological evolution.

$$p_{f,r} = \phi^0_0 p_f.$$

A value below one gives a smaller reference-expressed fermic scale. For the ideal clock held at rest in that chapter, the reference cycling rate falls by the same factor. The enlarged minor cycle is an illustration of the intended internal mechanism; the momentum and clock relations do not, by themselves, determine a unique physical radius.

A falling carrier must also obey its momentum budget. In the declared stationary free-fall comparison, its reference core M_r remains fixed while the reference-expressed fermic contribution falls and the bosic contribution grows. The torus's internal change therefore belongs beside a changing translation history. It is not a loss of core momentum along that no-exchange trajectory. Stopping and settling the carrier is a separate interaction that transfers momentum to its surroundings.

6.5.2 Bosonic dilation and expansion

The Expansion chapter changes a different part of the organization. For a supplied homogeneous stage history, the factor χ dresses the bosic contribution while the fermic scale remains fixed:

$$M_\chi = \sqrt{p_f^2 + \chi^2 p^2}.$$

The conserved spatial momentum label is still p ; the basic contribution to this generator is expressed as χp . In the inverse-length picture, decreasing χ corresponds to a longer bosonic cycle with the same number of repetitions. The major-radius change in Figure 6.4 illustrates that distinction: the bosonic geometry changes while the fermionic scale is held fixed.

Translation must follow the dressed generator as well. In the Expansion chapter's true-frame coordinates, motion along the chosen direction obeys

$$\dot{x}_k = \chi^2 c \frac{p_k}{M_\chi}.$$

This is the coordinate translation rate; an observer using bound-frame rods expresses the motion in the corresponding local units. At the same conserved p and fermic scale p_f , a smaller positive χ gives a lower forward coordinate speed. The carrier advances less during an equal reference-time interval, even though the bosonic cycle is represented as larger.

The two comparisons isolate different aspects of translation. Figure 6.3 follows a steady cycle and marks the advance per turn. Figure 6.5 changes the bosonic geometry while holding momentum content fixed, then compares advance per unit time. Assigning a helix to the second comparison would additionally require the basic cycle's period at each stage. Once that cadence is specified, the pitch follows from the same distance-per-cycle relation.

The two dilation pictures thus distinguish a gravitational change in the realized fermic scale from an expansion-driven change in the basic contribution. Each changes a different part of the internal organization, with its effect governed by the corresponding momentum relations.

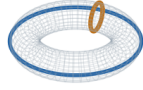
6.6 Resonant Loops and Waves

The closed path traced on the torus is a line. The surface provides a way to place its two phase coordinates, but the path itself can be followed without filling the surface. Seen this way, the resonance gives us a first string-like picture: a continuous loop winding through the two selected cycles.

There are two different changes we can now imagine. A marker can travel around the loop, recording progress through the resonant cycle. A pattern can also vary *along* the loop. To make the second idea visible, assign a scalar wave amplitude to each position and draw it as a colour. A crest is then a region of high amplitude, not a bulge in the torus surface.

Because the loop closes, a continuous scalar pattern must agree where its end rejoins its beginning. In the simple periodic example of Figure 6.6, a whole number of wave repetitions fits around the loop. That number labels a *mode*. Stretching the loop while keeping the mode fixed increases the distance occupied by each repetition. This is the wave picture behind the inverse-length momentum interpretation used for dilation.

Reference bosonic geometry



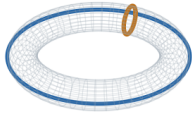
Reference major scale

Same momentum content; reference response



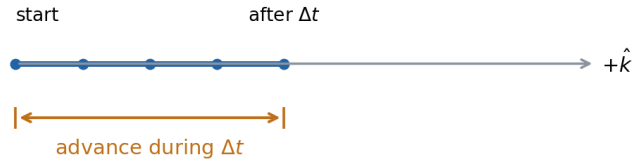
Dots: equal reference-time steps

Dilated bosonic geometry



Larger major scale; same minor scale

Same momentum content; reduced translation



Dots: equal reference-time steps

Matched p and p_f ; common internal display scale and external distance scale.

Figure 6.5: Bosonic dilation and translation at matched momentum content. The left panels show the reference torus and a torus with a larger major scale but the same minor scale. The right panels compare advance during equal intervals of true-frame coordinate time, keeping p and p_f fixed. The lower row uses a smaller positive χ and the stage-dressed motion law, giving a shorter advance. Dots mark equal time steps, not equal phase steps. The torus-size change uses the illustrative inverse-length interpretation; the displacement comes from the motion law.

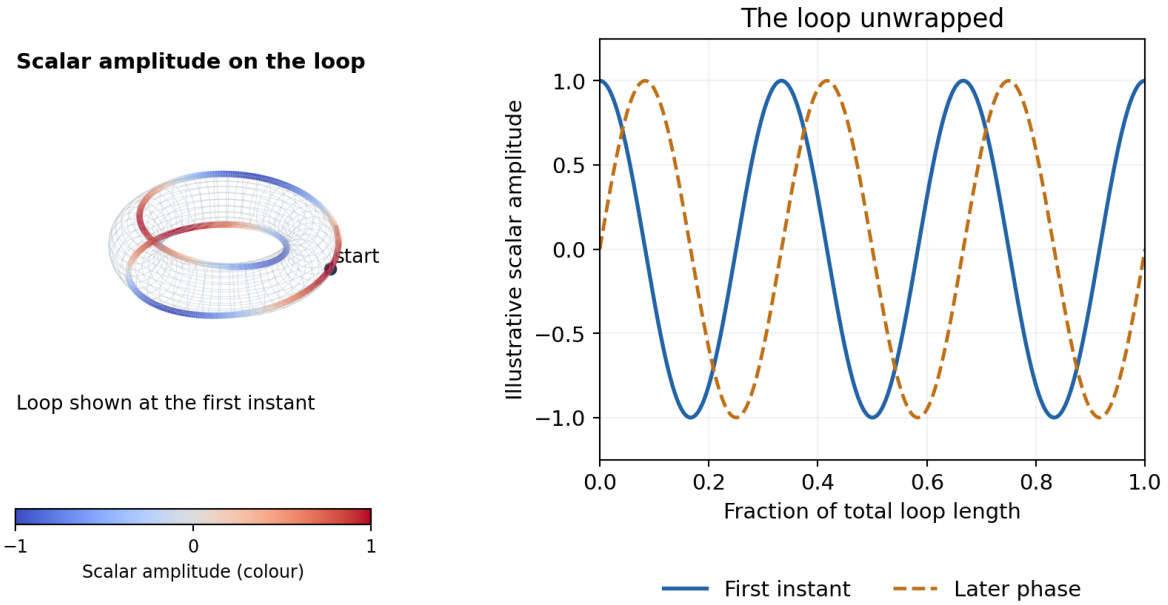


Figure 6.6: An illustrative wave on the selected resonant loop. Colour represents a scalar amplitude distributed along the path, not momentum magnitude or surface displacement. The right panel unwraps that same loop and shows an example pattern at two instants; it contains three full repetitions per circuit. Moving the pattern changes its phase while leaving the underlying path unchanged. The phase advance and mode count are illustrative choices.

We can also make a mode visible through the shape itself. A deformation can vary around the major ring, making some tube sections broader and others narrower. It can also vary around the minor cycle, changing the shape of each section. The second variation makes the fermic amplitude visible across the tube rather than only around the ring. In the standing pattern below, both variations oscillate in place: the surface passes through its undeformed form, then the outward and inward displacements exchange.

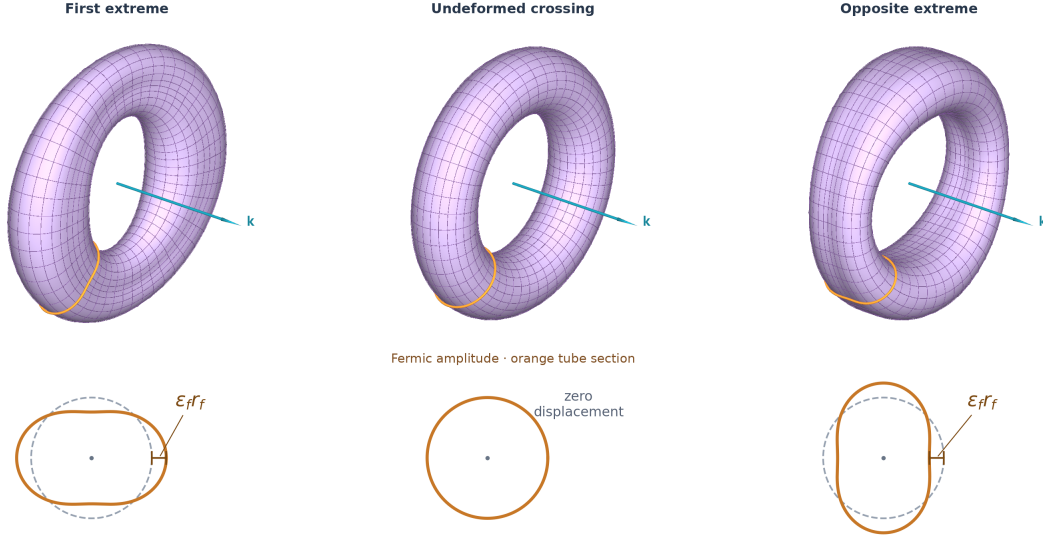


Figure 6.7: A standing deformation with variation around both torus cycles. Each lower panel shows the orange tube section above it, viewed face-on: the solid curve is the deformed section and the dashed circle its undeformed reference. At this selected section the major-ring contribution vanishes, exposing the minor-cycle contribution alone. Its maximum displacement is $\varepsilon_f r_f$, where r_f is the reference tube radius and ε_f sets the fractional geometric amplitude. The middle panel is the undeformed crossing between opposite extremes. The reference radii, camera and scale are fixed. This prescribed shape illustrates a mode; it is not a physical eigenmode derived from M1 dynamics.

Unlike the colour pattern on the fixed loop, Figure 6.7 changes the geometry. Unlike the uniform dilation shown earlier, it makes one region larger while another becomes smaller. These are different questions about a cycle: how its overall scale changes, what pattern it carries, and whether that pattern deforms its shape.

A physical wave equation must determine which patterns propagate and which survive disturbances. More structured fields can also carry orientation information around the loop. Their return conditions need more than the single scalar amplitude used in these illustrations.

The same distinction matters for particle identity. A persistent particle would require an allowed internal organization that remains stable and has definite momentum and interaction properties.

A winding number is a useful label for a pattern; it is not, on its own, an identification of that pattern with an electron or another species.

The loop prepares an equally useful question for string theory. Can an extended, phase-bearing object supply the dynamics that the torus picture leaves unspecified? To answer, we will have to compare what the object carries, what its allowed modes are and how it interacts. Calling the drawn loop a string is the beginning of that comparison, not its conclusion.

6.7 Beyond the First Picture

The torus has joined several relationships in one picture. Its two cycles keep the fermic and bosic momentum roles visible. Its resonant path shows internal return; the helix records translation during that return. Its two radii distinguish illustrative fermionic and bosonic dilation, and a pattern on the closed path gives waves somewhere to live.

Yet the same path can carry different wave patterns. Even a specified pattern can carry information the drawing has not shown: an orientation, a relative phase between components, or a rule for comparing it with another carrier. We can picture the first omission by attaching a small arrow to our moving marker. The marker may return to its starting point while the arrow has changed its orientation. Position on the torus no longer specifies the whole state.

Interactions make this additional information consequential. Two carriers can respond differently when their internal phases or orientations differ, even if their displayed radii and winding paths agree. A geometry capable of describing them must say how those states are compared across space and how their momentum changes through coupling. A bound rod or a material clock also requires such interactions; its behavior cannot be inferred by stretching a single torus drawing.

The next step keeps the two-cycle picture while adding the phase and orientation information needed to describe a fuller internal state. The Clifford torus provides a different geometric setting for two periodic coordinates, while Clifford algebra supplies the first-order representation of the momentum shell already encountered in quantum mechanics. Understanding their respective roles will let us ask how recurrence, orientation and translation can be described together—and what further dynamics makes that description a physical model.

Part IV

Appendices

Derivation 2.3A — Structural uniqueness of the directional shell readings

Purpose

This derivation supports the claim, referenced from Foundations §2.3, that the realized directional shell readings

$$p_k^+ = M + \frac{1}{2}p_k, \quad p_k^- = M - \frac{1}{2}p_k,$$

are not an arbitrary choice within the natural class of additive, symmetry-compatible axis-wise two-reading encodings.

The result proved here is deliberately precise in scope. We show that, under mild regularity, additivity and directional-reflection symmetry force the two readings to take an affine form

$$F_{\pm}(M, p) = \alpha M \pm \beta p,$$

and that the canonical M1 realization is obtained by the normalization that identifies the recovered conserved quantities with the already named M and p_k . Here F_+ and F_- are proof labels for evaluations in opposed orientations, not new physical momentum variables.

This is therefore an **affine uniqueness theorem**. It does **not** prove that every imaginable nonlinear realization of the directional shell-reading structure is impossible.

It also does not cover quadratic, flux-like source-channel constructions such as null contractions of a rank-two stress tensor. Those live in a different mathematical class: this appendix constrains linear charge-like readings of conserved vector content — a class that the gravitational carrier source map of Derivation 3.3A sits squarely inside, being the shell readings themselves densitized — while the rejected null-probe diagnostic of Derivation 3.3B works with quadratic stress-tensor contractions.

Role / regime / status

- **Role:** derivation
- **Regime:** isolated systems, inertial kinematic level, one fixed chosen orientation $\hat{k}$
- **Status:** proved within the additive, symmetry-compatible, regular axis-wise class stated below
- **Unresolved stronger claim:** global uniqueness beyond the affine/additive class remains open

Setup

Fix an oriented unit direction $\hat{k}$. For readability write

$$p := p_k.$$

Consider an axis-wise two-reading encoding that assigns to each pair (M, p) two real numbers

$$F_+(M, p), \quad F_-(M, p),$$

interpreted as candidate evaluations in the orientations $\hat{k}$ and $-\hat{k}$. They are placed side by side here to test invertibility; they are not two simultaneous particle contributions within one ADMC reading.

Define the sum and difference combinations

$$\Sigma(M, p) := F_+(M, p) + F_-(M, p),$$

$$\Delta(M, p) := F_+(M, p) - F_-(M, p).$$

For the canonical M1 realization these become

$$\Sigma = 2M, \quad \Delta = p.$$

The aim is to determine how much freedom is left if the encoding behaves like a genuine additive directional bookkeeping map rather than an arbitrary relabeling.

Assumptions

Assumption A1: axis-wise additivity

For each fixed orientation and each sign label,

$$F_{\pm}(M_1 + M_2, p_1 + p_2) = F_{\pm}(M_1, p_1) + F_{\pm}(M_2, p_2)$$

for all pairs (M_1, p_1) and (M_2, p_2) in the domain.

Equivalently, Σ and Δ are additive.

This is the structural assumption that the shell readings are true additive bookkeeping variables for isolated-system composition.

Assumption A2: parity exchange

Reversing the sign of the directional component exchanges the two orientation labels:

$$F_+(M, -p) = F_-(M, p), \quad F_-(M, -p) = F_+(M, p).$$

Equivalently, $\Sigma(M, p)$ is even in p and $\Delta(M, p)$ is odd in p .

Assumption A3: axis-independence

The functional form of the encoding does not depend on which axis is chosen. Thus the same functions $F_{\pm} : \mathbb{R}^2 \rightarrow \mathbb{R}$ apply for every oriented axis.

Assumption A4: local invertibility

There exists at least one point (M_0, p_0) with $M_0 > 0$ such that the map

$$(M, p) \mapsto (F_+(M, p), F_-(M, p))$$

is locally one-to-one in a neighborhood of (M_0, p_0) .

This assumption excludes degenerate encodings that collapse two degrees of freedom into one.

Assumption A5: mild regularity

At least one of the additive functions F_+ , F_- — equivalently Σ or Δ — is measurable on a set of positive measure, or continuous at one point.

This is a standard regularity hypothesis used to exclude pathological solutions to the Cauchy functional equation (Kuczma 2009).

Main theorem

Theorem 1 (affine form is forced)

Under Assumptions A1–A5, there exist constants $\alpha, \beta \in \mathbb{R}$ such that

$$F_{\pm}(M, p) = \alpha M \pm \beta p$$

for all (M, p) in the domain.

Moreover, Assumption A4 implies

$$\alpha \neq 0, \quad \beta \neq 0.$$

Proof

By Assumption A1, each of F_+ and F_- satisfies the additive functional equation on $\mathbb{R}^2$:

$$F_{\pm}((M_1, p_1) + (M_2, p_2)) = F_{\pm}(M_1, p_1) + F_{\pm}(M_2, p_2).$$

By Assumption A5, additive functions on $\mathbb{R}^2$ with this mild regularity are linear. Therefore there exist constants $a_{\pm}, b_{\pm} \in \mathbb{R}$ such that

$$F_{\pm}(M, p) = a_{\pm}M + b_{\pm}p.$$

Now impose Assumption A2. From parity exchange,

$$F_+(M, -p) = F_-(M, p)$$

for all (M, p) . Substituting the linear forms gives

$$a_+M - b_+p = a_-M + b_-p.$$

Because this identity holds for all M and p , coefficient comparison yields

$$a_+ = a_- =: \alpha, \quad b_- = -b_+ =: -\beta.$$

Hence

$$F_{\pm}(M, p) = \alpha M \pm \beta p.$$

It remains to use local invertibility. If $\alpha = 0$, then

$$F_+(M, p) + F_-(M, p) \equiv 0,$$

so the reading pair cannot recover any variation in M ; the map cannot be locally one-to-one. If $\beta = 0$, then

$$F_+(M, p) - F_-(M, p) \equiv 0,$$

so the pair cannot recover any variation in p ; again local invertibility fails. Therefore $\alpha \neq 0$ and $\beta \neq 0$.

Interpretation of Theorem 1

Theorem 1 is the core structural result. It says that once the encoding is required to be

- additive,
- compatible with directional reversal,
- axis-independent,
- locally invertible,
- and regular enough to exclude pathological additive functions,

then no nonlinear or asymmetrically mixed dependence on (M, p) survives. The entire admissible class collapses to the two-parameter affine family

$$F_{\pm}(M, p) = \alpha M \pm \beta p.$$

At this stage the coefficients are still undetermined. That remaining freedom is normalization freedom, not structural freedom.

Corollary 1 (normalization class)

Under the assumptions of Theorem 1, every admissible encoding is of the form

$$F_{\pm}(M, p) = \alpha M \pm \beta p$$

with $\alpha \neq 0$ and $\beta \neq 0$.

Consequently,

$$\frac{1}{2}(F_+ + F_-) = \alpha M, \quad F_+ - F_- = 2\beta p.$$

So the affine family contains a global scaling freedom for the sum combination and an independent relative scaling for the difference combination.

Proof

Substitute the form from Theorem 1 directly into the sum and difference combinations.

Canonical normalization

To recover the canonical M1 variables without rescaling, one imposes the normalization

$$M = \frac{1}{2}(F_+ + F_-), \quad p = F_+ - F_-.$$

Substituting the affine family into these identities gives

$$\alpha M = M, \quad 2\beta p = p,$$

hence necessarily

$$\alpha = 1, \quad \beta = \frac{1}{2}.$$

This yields the canonical directional shell readings.

Corollary 2 (canonical realization)

If, in addition to Assumptions A1–A5, one requires that the recovered additive quantities are exactly the already named variables M and p , namely

$$M = \frac{1}{2}(F_+ + F_-), \quad p = F_+ - F_-,$$

then the encoding is uniquely fixed as

$$F_{\pm}(M, p) = M \pm \frac{1}{2}p.$$

Equivalently, reinstating the directional component and physical reading labels,

$$p_k^+ = M + \frac{1}{2}p_k, \quad p_k^- = M - \frac{1}{2}p_k.$$

Proof

This is the canonical normalization just derived, written as a uniqueness statement.

Positivity and sign convention

If one additionally requires

$$F_{\pm}(M, p) \geq 0$$

on the physical domain, then positivity does not determine the coefficients by itself, but it does select the physical sign convention.

For the canonical normalization,

$$F_{\pm}(M, p) = M \pm \frac{1}{2}p,$$

so positivity is automatic on the physical domain $M \geq |p|$. The sign of the $+$ reading is fixed by the convention that increasing p along the chosen orientation increases F_+ and decreases F_- .

What has and has not been proved

The status of the result should be stated carefully.

Proved here

Within the additive, symmetry-compatible, axis-wise class defined by Assumptions A1–A5,

1. the reading encoding must be affine,
2. local invertibility removes the degenerate cases $\alpha = 0$ and $\beta = 0$,
3. canonical normalization uniquely fixes the coefficients to $\alpha = 1$ and $\beta = 1/2$.

Not proved here

This derivation does **not** establish that every sufficiently reasonable realization of the directional shell-reading structure must satisfy Assumption A1 from first principles. It also does **not** exclude every conceivable nonlinear encoding outside the additive class.

So the honest conclusion is:

the canonical directional shell readings are structurally unique within the natural additive and symmetry-compatible axis-wise encoding class.

That is already a strong result, but it is weaker than a global no-alternative theorem.

Result

The directional shell readings used in M1,

$$p_k^+ = M + \frac{1}{2}p_k, \quad p_k^- = M - \frac{1}{2}p_k,$$

are the unique canonically normalized members of the affine family forced by axis-wise additivity, parity exchange, axis-independence, local invertibility, and mild regularity.

For a technically strong reader outside the project, the correct interpretation is therefore not that the form was guessed and then merely preferred, but that it is tightly constrained once one decides that directional shell readings are genuine additive conserved quantities attached to a chosen orientation.

Notes for citation in the main text

If Foundations §2.3 cites this appendix, the safest summary sentence is:

Within the natural additive and symmetry-compatible class of axis-wise opposed-reading encodings, the shell readings are forced to the affine family $F_{\pm} = \alpha M \pm \beta p$, and canonical normalization fixes uniquely the realized form $p_k^+ = M + \frac{1}{2}p_k$ and $p_k^- = M - \frac{1}{2}p_k$.

Derivation 3.3A — Resolved Gravitational Sources

Purpose

A local particle population carries core content, directional imbalance and momentum transport. This derivation constructs these parts of the source before averaging, shows what the opposed channels retain, and separates the transport trace from the spacetime trace. It supplies the kinetic source used in §3.3.

Starting assumptions

For §3.3's weak-field source approximation, take the particles to have their flat-space momentum configurations. The kernel uses their actual local core and spatial momentum; changing a particle configuration changes those inputs, not the definition of the source.

Work at one event in a future local orthonormal source frame. Greek indices run from 0 to 3, Latin spatial indices from 1 to 3; carrier and species labels are summed explicitly. Use

$$\eta_{\mu\nu} = \text{diag}(-1, 1, 1, 1), \quad x^0 = ct.$$

The opposite signature sometimes used for the Foundations SR interval reverses the signed contraction, not the physical positive core or opposed readings. Here the future shell has contraction $-p_{f,a}^2$, and the local covector has temporal component $-M_a$. In an opposite-signature convention the contraction and metric-lowered covector both reverse sign and must be converted together.

The free-carrier shell used here is

$$p_a^\mu = (M_a, p_a^i), \quad p_{\mu,a} = (-M_a, p_{i,a}), \quad M_a = \sqrt{p_{f,a}^2 + \delta_{ij} p_a^i p_a^j} > 0. \quad (6.1)$$

$p_{f,a}$ is the fermic momentum of particle a in its chosen free-space reference realization, held fixed in this free-carrier coupling. Nonzero null carriers have $p_{f,a} = 0$; the zero null covector is excluded. All momentum components, M_a and $p_{f,a}$ have momentum units.

The brackets denote a positive linear moment per physical volume, at the same event. A discrete local stream representation is

$$\left\langle \sum_a A_a \right\rangle = \sum_a w_a A_a, \quad w_a \geq 0,$$

where w_a has number-per-volume units. For a continuous species population we adopt a nonnegative scalar shell distribution F_α and define

$$\left\langle \sum_a A_a \right\rangle = \sum_\alpha \int F_\alpha(x, \mathbf{p}) A_\alpha(\mathbf{p}) d^3p. \quad (6.2)$$

$F_\alpha d^3p$ is the number density in this local frame; F_α has inverse-volume per momentum-cubed units. Assume the required moments are finite. This pointwise kinetic construction also applies in a curved geometry when all inputs and physical-volume weights belong to the same local frame; its weak-field use in §3.3 does not make the finite source a sum of remote coordinate components.

Derivation

Opposed readings and the resolved kernel

For a specified unit axis covector $n_i^{(k)}$, define

$$p_{k,a} = n_i^{(k)} p_a^i, \quad p_{k,a}^\pm = M_a \pm \frac{1}{2} p_{k,a}.$$

Because $|p_{k,a}| \leq |\mathbf{p}_a| \leq M_a$, each reading is at least $M_a/2 > 0$. Their inverse map is

$$M_a = \frac{p_{k,a}^+ + p_{k,a}^-}{2}, \quad p_{k,a} = p_{k,a}^+ - p_{k,a}^-. \quad (6.3)$$

The resolved kernel and aggregate are

$$j_a^{\mu\nu} = \frac{p_a^\mu p_a^\nu}{M_a}, \quad J^{\mu\nu} = \left\langle \sum_a j_a^{\mu\nu} \right\rangle = \begin{bmatrix} \mathcal{M} & \mathcal{P}^j \\ \mathcal{P}^i & \mathcal{C}^{ij} \end{bmatrix}, \quad (6.4)$$

with

$$\mathcal{M} = \left\langle \sum_a M_a \right\rangle, \quad \mathcal{P}^i = \left\langle \sum_a p_a^i \right\rangle, \quad \mathcal{C}^{ij} = \left\langle \sum_a \frac{p_a^i p_a^j}{M_a} \right\rangle.$$

The single-particle contribution j_a has momentum units; averaging gives J momentum-per-volume units. Its temporal entry is the positive core content, and its time-space entries are the signed momentum components. Summing the opposed channels gives

$$\mathcal{J}_k^\pm = \mathcal{M} \pm \frac{1}{2} n_i^{(k)} \mathcal{P}^i.$$

Why the measure is part of the tensor statement

At a fixed event $p_{f,\alpha}$ is a scalar. Integrating the invariant future-shell measure over its temporal momentum gives

$$2\mathbf{1}_{p^0>0} \delta\left(-(p^0)^2 + |\mathbf{p}|^2 + p_{f,\alpha}^2\right) d^4p = \frac{d^3p}{M_\alpha}.$$

For a proper future-preserving Lorentz change Λ , $M'_a = p_a'^0$ generally differs from M_a . Consequently

$$j'_a = \frac{M_a}{M'_a} \Lambda j_a \Lambda^\top, \quad F'_\alpha d^3p' = \frac{M'_\alpha}{M_\alpha} F_\alpha d^3p. \quad (6.5)$$

The factors cancel in the aggregate, giving $J' = \Lambda J \Lambda^\top$. For discrete streams the density changes as $w'_a = (M'_a/M_a)w_a$. Both momenta and density weights therefore transform under a boost. Spatial rotations leave M_a unchanged.

Now relate the local and coordinate momentum measures. In a slice-normal coframe, $\phi^0_i = 0$, $\phi^0_0 > 0$, choose its spatial orientation positive. Use $P_m = p_{\text{coord},m}$ and $p_i = p_{\text{local},i}$ for this calculation. Coframe duality gives

$$P_m = \phi^i_m p_i, \quad d^3P = \det[\phi^i_m] d^3p, \quad d\Sigma = \det[\phi^i_m] d^3x.$$

Thus $d^3x d^3P = d\Sigma d^3p$. Furthermore,

$$p_{\text{coord}}^0 = \frac{M}{\phi^0_0}, \quad \frac{d^3P}{\sqrt{-g} p_{\text{coord}}^0} = \frac{d^3p}{M}, \quad \sqrt{-g} = \phi^0_0 \det[\phi^i_m]. \quad (6.6)$$

The denominator uses contravariant p_{coord}^0 , not the coordinate generator $-p_{\text{coord},0}$ or the scaled local component p_r^0 of §3.5. These measure relations also hold pointwise for a specified

scalar local shell. Conservation of this distribution under evolution requires the actual carrier equations; covariance of its instantaneous moment alone proves no conservation law.

Symmetry and transport correlations

The two momentum factors in Equation 6.4 give $J^{\mu\nu} = J^{\nu\mu}$. For any real covector z_μ , the same construction gives

$$z_\mu J^{\mu\nu} z_\nu = \left\langle \sum_a \frac{(z_0 M_a + z_i p_a^i)^2}{M_a} \right\rangle \geq 0. \quad (6.7)$$

Lowered forms follow from the local metric:

$$J^\mu{}_\nu = J^{\mu\alpha} \eta_{\alpha\nu}, \quad J_{\mu\nu} = \eta_{\mu\alpha} \eta_{\nu\beta} J^{\alpha\beta} = J_{\nu\mu}.$$

The mixed array reverses the temporal column and need not be symmetric or positive semidefinite. This is an index conversion, not a change of physical content.

Write the local velocities in units of c as $v_a^i = p_a^i / M_a$. For $\mathcal{M} > 0$, their content-weighted mean is $\bar{v}^i = \mathcal{P}^i / \mathcal{M}$. Expanding the transport around this mean isolates the information absent from the summed channels:

$$\mathcal{C}^{ij} = \frac{\mathcal{P}^i \mathcal{P}^j}{\mathcal{M}} + \left\langle \sum_a M_a (v_a^i - \bar{v}^i)(v_a^j - \bar{v}^j) \right\rangle. \quad (6.8)$$

The cross terms vanish because $\langle \sum_a M_a (v_a - \bar{v}) \rangle = 0$. Contracting the last term with a spatial covector twice gives a sum of nonnegative squares. This contribution is positive semidefinite and vanishes only when all populated local velocities coincide. At zero content, all moments vanish.

The same positive weights give $|\mathcal{P}| \leq \langle \sum_a |\mathbf{p}_a| \rangle \leq \mathcal{M}$, the local content–direction cone bound.

For two equal streams of weight w and opposite momenta $\pm p e_1$,

$$\mathcal{M} = 2wM, \quad \mathcal{P}^i = 0, \quad [\mathcal{C}^{ij}] = \frac{2wp^2}{M} \text{diag}(1, 0, 0), \quad M = \sqrt{p_f^2 + p^2}.$$

Replacing e_1 by e_2 leaves every summed opposed channel unchanged but rotates the nonzero transport array. The channels therefore leave out the direction of momentum transport. In particular, $n_i \mathcal{C}^{ij} n_j$ detects a correlation erased by $\mathcal{P}^i = 0$.

Core content, transport and the source trace

Contraction of the free-carrier shell gives

$$-\eta_{\mu\nu}J^{\mu\nu} = \mathcal{M} - \delta_{ij}\mathcal{C}^{ij} = \left\langle \sum_a \frac{p_{f,a}^2}{M_a} \right\rangle \geq 0. \quad (6.9)$$

Core content $\mathcal{M}$ sources the common deformation θ^{00} in the weak field. The transport trace remains in the spatial deformation block and contributes to the temporal map through $\phi^0_0 \simeq 1 - \theta^{00} - \delta_{kl}\theta^{kl}$. Its temporal effect therefore involves $\mathcal{M} + \delta_{ij}\mathcal{C}^{ij}$, whereas the spacetime trace above contains their difference.

Cold carriers at rest have $\mathcal{C} = 0$, so both combinations coincide with $\mathcal{M}$. For nonzero null carriers, $\delta_{ij}\mathcal{C}^{ij} = \mathcal{M}$: the spacetime trace vanishes while the weak temporal-readout combination is $2\mathcal{M}$. An isotropic null population has $\mathcal{P} = 0$ and $\mathcal{C}^{ij} = (\mathcal{M}/3)\delta^{ij}$. Neither its zero net direction nor its zero spacetime trace erases its gravitational source.

Result

The resolved kernel supplies three distinct source blocks. Opposed channels determine content and directional imbalance; the transport tensor retains additional carrier correlations. Positive physical-volume weights give the stated symmetry, cone and covariance properties. The kinetic spacetime trace differs from the combination governing weak temporal readout.

These blocks provide the kinetic source used in §3.4.

Notes

Extending this local moment to a finite spatial window requires a comparison map between tangent spaces, a slice, evolving weights and boundary fluxes. Derivation 3.7A develops the corresponding balance conditions.

Binding, interfaces and supports require material source laws; their stresses need not admit a positive free-carrier decomposition. The kinetic calculation leaves these additional contributions and full isolated-system ADMC open.

Derivation 3.4A — Sourced Spatial Deformation

Purpose

This derivation supports §3.4's source-to-deformation step. It constructs the operational map from the registered deformation, obtains the free-carrier source from its action, and gives the controlled weak exterior used in the later comparisons.

Starting assumptions

Use the source convention of Derivation 3.3A, with signature $(-+++)$ and $x^0 = ct$. The map connects coordinate inputs to local outputs:

$$g_{\mu\nu} = \eta_{\alpha\beta} \phi^\alpha{}_\mu \phi^\beta{}_\nu, \quad d\ell^\alpha = \phi^\alpha{}_\mu dx^\mu.$$

The metric field law below is a specified physical input. Together with the free-carrier action it defines the comparison sector calculated here. It does not derive the internal redistribution of a structured M1 particle or introduce an independent internal-modulus field. Greek indices on coordinate momenta are raised with g ; in the local frame, $p^0 = M$ and $M^2 = p_f^2 + |\mathbf{p}|^2$.

Derivation

Registered deformation coordinates

The deformation has the symmetric component form

$$[\theta^{\mu\nu}] = \begin{bmatrix} \theta^{00} & \theta^{0j} \\ \theta^{i0} & \theta^{ij} \end{bmatrix}, \quad \theta^{i0} = \theta^{0i}, \quad \theta^{ij} = \theta^{ji}. \quad (6.10)$$

The ten independent entries parametrize the geometry. Lowering a registered index uses the reference form, $\theta_{\mu\nu} = \eta_{\mu\alpha} \eta_{\nu\beta} \theta^{\alpha\beta}$; coordinate indices instead use the varying metric.

The registered metric dictionary is

$$g_{00} = -1 + 2(\theta^{00} + \delta_{kl}\theta^{kl}), \quad g_{0i} = -4\delta_{ij}\theta^{0j}, \quad g_{ij} = (1 + 2\theta^{00} - 2\delta_{kl}\theta^{kl})\delta_{ij} + 4\delta_{ik}\delta_{jl}\theta^{kl}. \quad (6.11)$$

With the reference form explicit, the same dictionary is

$$g_{\mu\nu} = \eta_{\mu\nu} + 4\eta_{\mu\alpha}\theta^{\alpha\beta}\eta_{\beta\nu} - 2\eta_{\mu\nu}\eta_{\alpha\beta}\theta^{\alpha\beta}.$$

The fixed time split and reference deltas belong to this dictionary. A tensor transformation carries that reference structure with θ . Extracting a fresh chart from a transformed metric instead reapplies the dictionary with the new reference split. Although the dictionary is affine, curvature built from g contains nonlinear products.

For construction only, set $h = [g_{mn}]$ and let $s = h_+^{1/2}$ be its symmetric positive-definite square root. Fix the local spatial orientation by

$$[\phi^i_m] = s.$$

The matrices h and s are computed from g . Choosing the positive square root fixes the local spatial axes for this chart. Complete the coframe by

$$\phi^0_m = 0, \quad \phi^i_0 = -4\delta^{ij}(\phi^{-1})^m_j\delta_{mn}\theta^{0n}, \quad \phi^0_0 = \sqrt{1 - 2(\theta^{00} + \delta_{kl}\theta^{kl}) + \delta_{ij}\phi^i_0\phi^j_0}. \quad (6.12)$$

Since $s^\top s = s^2 = h$, the spatial Gram matrix reproduces g_{mn} . Multiplying the second expression by $\delta_{ij}\phi^i_m$ gives $g_{m0} = -4\delta_{mn}\theta^{0n}$; the last expression gives g_{00} in Equation 6.11. The reconstructed ϕ therefore reproduces every block of g .

To display the inverse, use $E = [\phi^i_m]$ and $v = [\phi^i_0]$ as temporary arrays. Then

$$[\phi^{-1}] = \begin{bmatrix} (\phi^0_0)^{-1} & 0 \\ -E^{-1}v/(\phi^0_0) & E^{-1} \end{bmatrix}, \quad g^{\mu\nu} = (\phi^{-1})^\mu_\alpha (\phi^{-1})^\nu_\beta \eta^{\alpha\beta}. \quad (6.13)$$

Both multiplication orders give identity. The admitted domain is positive-definite h , positive radicand and the future root $\phi^0_0 > 0$. A coordinate-static timelike worldline additionally requires $1 - 2(\theta^{00} + \delta_{kl}\theta^{kl}) > 0$.

The configuration can also be recovered. Put $d_{ij} = g_{ij} - \delta_{ij}$ and use the temporary trace helper $t_\theta = [3(g_{00} + 1) - \delta^{ij}d_{ij}]/8$. Then

$$\theta^{00} = (g_{00} + 1 + \delta^{ij}d_{ij})/8, \quad \theta^{0i} = -\delta^{ij}g_{0j}/4, \quad \theta^{ij} = \frac{1}{4}(\delta^{ik}\delta^{jl}d_{kl} - 2\theta^{00}\delta^{ij} + 2t_\theta\delta^{ij}),$$

and taking the spatial trace proves $t_\theta = \delta_{ij}\theta^{ij}$.

Subsequent frame changes carry the coframe and source together; the transformed spatial coframe need not remain symmetric. The temporal–spatial components and compatible frame transport remain part of the reconstructed geometry.

Free-carrier response and source

For a carrier with fixed free-space reference momentum $p_{f,a}$, use the constrained action

$$S = \sum_a \int \left[p_{\mu,a} \dot{x}_a^\mu - \frac{e_a}{2} \left(g^{\mu\nu} p_{\mu,a} p_{\nu,a} + p_{f,a}^2 \right) \right] d\lambda. \quad (6.14)$$

A dot denotes differentiation with respect to the trajectory parameter λ ; e_a enforces the local shell. The action has momentum-times-length units. Variation gives the shell, trajectory and momentum equations developed in Derivation 3.5A.

The metric source comes from varying this same action. Since

$$\delta g^{\mu\nu} = -g^{\mu\alpha} g^{\nu\beta} \delta g_{\alpha\beta},$$

its variation is

$$\delta S = \frac{1}{2} \sum_a \int e_a p_a^\alpha p_a^\beta \delta g_{\alpha\beta}(x_a) d\lambda.$$

Thus the particle source, in coordinate components, is

$$J^{\alpha\beta}(x) = \sum_a \int e_a p_a^\alpha p_a^\beta \frac{\delta^{(4)}(x - x_a)}{\sqrt{-g}} d\lambda. \quad (6.15)$$

On a local time slice, $\ell^0 = e_a M_a$. Eliminating the trajectory delta function supplies the factor $1/M_a$, recovering §3.3's particle source in local components:

$$J^{\mu\nu} = \left\langle \sum_a \frac{p_a^\mu p_a^\nu}{M_a} \right\rangle.$$

For a material system, the variation also includes its internal, binding and support terms. Their contributions enter the same source J .

The field law and source data

Choose the metric action, in the same momentum-length units,

$$S_g = \frac{c^3}{16\pi G_N} \int R(g) \sqrt{-g} d^4x.$$

With the appropriate boundary term or boundary conditions, its variation together with the consistently derived source gives

$$G^{\mu\nu}(g) = \frac{8\pi G_N}{c^3} J^{\mu\nu}. \quad (6.16)$$

Both tensors are in coordinate components, with curvature indices raised using g . The local source converts by $J_{\text{coord}} = \phi^{-1} J_{\text{local}} \phi^{-\text{T}}$. A solution requires compatible source dynamics, initial or boundary data, and the reference registration used to reconstruct ϕ .

In the aligned weak-field expansion, the harmonic condition $\partial_\mu \theta^{\mu\nu} = 0$ reduces the linearized curvature to

$$G^{(1)\mu\nu} = -2\Box\theta^{\mu\nu}, \quad \Box = -\partial_0^2 + \delta^{ij}\partial_i\partial_j.$$

The field equation therefore gives

$$\Box\theta^{\mu\nu} = -\frac{4\pi G_N}{c^3} J^{\mu\nu}. \quad (6.17)$$

For a localized, conserved stationary leading source, $\Box$ reduces to the spatial Laplacian. Its decaying Green solution is the componentwise integral in §3.4, with the same positive coupling for every source component.

A common weak exterior approximation

Supply a stationary spherical source of radius R_s , dominated by cold, slowly moving carriers. Binding, internal transport and support corrections are small at the retained order, but the source is understood to include the stresses that maintain its configuration. Define its leading integrated core and length coefficient by

$$M_{\text{src}} = \int \mathcal{M} d^3x, \quad \ell_g = \frac{G_N M_{\text{src}}}{c^3}. \quad (6.18)$$

Use a weak source, $\ell_g/R_s \ll 1$, and the distant normalization $\phi^0_0 \rightarrow 1$, $\phi^i_j \rightarrow \delta^i_j$. In the weak stationary source law of §3.4,

$$\Delta\theta^{00} = -\frac{4\pi G_N}{c^3}\mathcal{M}.$$

Because $\Delta(1/|\mathbf{x} - \mathbf{y}|) = -4\pi\delta^{(3)}(\mathbf{x} - \mathbf{y})$, the decaying solution is

$$\theta^{00}(\mathbf{x}) = \frac{G_N}{c^3} \int \frac{\mathcal{M}(\mathbf{y})}{|\mathbf{x} - \mathbf{y}|} d^3y.$$

Spherical symmetry reduces the exterior integral to ℓ_g/r . The directional and spatial source moments are subleading in this cold-source approximation. The weak map therefore gives

$$\begin{aligned} \phi^0_0 &\simeq 1 - \frac{\ell_g}{r}, & \phi^i_j &\simeq \left(1 + \frac{\ell_g}{r}\right) \delta^i_j, \\ g_{00} &\simeq -1 + \frac{2\ell_g}{r}, & g_{ij} &\simeq \left(1 + \frac{2\ell_g}{r}\right) \delta_{ij}. \end{aligned} \tag{6.19}$$

These expressions retain first order in the exterior deformation and the leading cold-source moments. Comparisons using this weak exterior take $r \gg R_s$, negligible probe backreaction, and source corrections smaller than the retained terms. Interior matching and support are inputs to this exterior calculation. Appreciable transport, binding or nonlinear deformation requires the corresponding source and field calculation.

Result

The registered deformation determines the operational map on the stated regular domain. The selected action reproduces the free-carrier kinetic source, while the retained metric response produces the weak spherical exterior used in §3.10.

Notes

The action supplies both free-carrier motion and its reciprocal source. A material extension must supply the constrained internal dynamics and derive its complete source from them. Full curved-system ADMC also requires a common directional comparison that includes deformation, support and boundary transport; Derivation 3.7A sets out this open task.

Derivation 3.5A — Particle Response and Internal Cycling

Purpose

A stationary deformation can preserve a particle's reference core while changing its local momentum and motion. This derivation obtains that result from the free-carrier action, constructs §3.5's directional shell from reference-expressed four-momentum, and derives the cycle rate under its explicit clock hypothesis.

Starting assumptions

Use the free-carrier action of Derivation 3.4A, with fixed free-space reference p_f , positive local M , and signature $(-+++)$. For the reference-root calculation take a zero-shift map,

$$\phi^0_i = \phi^i_0 = 0, \quad \phi^0_0 > 0, \quad g_{ij} = \delta_{kl} \phi^k_i \phi^l_j.$$

The inverse spatial block is g^{ij} . Momentum indices refer to the stated frame; when local and coordinate spatial momenta occur together, the local ones are labelled explicitly. Work in the test-particle limit with a prescribed background.

For an ideal massive clock, assume that a physical phase follows the action according to $d\varphi = -dS/\hbar$, with one recurrence per 2π phase increment. This cycle coupling is an additional model premise.

Derivation

Carrier variation and the reference root

Variation of the constrained action with respect to the multiplier, momentum and position gives

$$\begin{aligned}
g^{\mu\nu}p_\mu p_\nu + p_f^2 &= 0, \\
\dot{x}^\mu &= e g^{\mu\nu}p_\nu, \\
\dot{p}_\gamma &= -\frac{e}{2}(\partial_\gamma g^{\mu\nu})p_\mu p_\nu.
\end{aligned} \tag{6.20}$$

The dot refers to the trajectory parameter λ . In the zero-shift branch the positive root is

$$-p_0 = \phi^0{}_0 M, \quad M = \sqrt{p_f^2 + g^{ij}p_i p_j}, \quad p_i = p_j^{\text{local}} \phi^j{}_i. \tag{6.21}$$

Eliminating the constraint and using x^0 as parameter gives the reduced action

$$S = \int \left[p_i \frac{dx^i}{dx^0} - (-p_0) \right] dx^0.$$

Hamilton's equations therefore differentiate the same positive root:

$$\begin{aligned}
\frac{dx^i}{dx^0} &= \frac{\partial(-p_0)}{\partial p_i} = \frac{\phi^0{}_0}{M} g^{ij} p_j, \\
\frac{dp_i}{dx^0} &= -\frac{\partial(-p_0)}{\partial x^i} = -M \partial_i \phi^0{}_0 - \frac{\phi^0{}_0}{2M} (\partial_i g^{jk}) p_j p_k.
\end{aligned} \tag{6.22}$$

At an instant of local rest, differentiation of the first equation gives

$$\frac{d^2 x^i}{(dx^0)^2} = -\phi^0{}_0 g^{ij} \partial_j \phi^0{}_0. \tag{6.23}$$

The term differentiating the velocity coefficient vanishes at that instant because $p_j = 0$. Acceleration per reference time $t = x^0/c$ is c^2 times this expression.

Four-momentum and the directional shell

For a massive carrier, define $u^\mu = dx^\mu/d\tau$ with $g_{\mu\nu}u^\mu u^\nu = -c^2$. The shell and trajectory relations in Equation 6.20 give

$$g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu = -e^2 p_f^2, \quad c \frac{d\tau}{d\lambda} = e p_f$$

on the future-directed branch. Consequently the local momentum is

$$(p^\mu)_{\text{local}} = \phi^\mu{}_\nu g^{\nu\gamma} p_\gamma = \frac{p_f}{c} \phi^\mu{}_\nu u^\nu.$$

Express every local component with the same distant normalization:

$$p_r^\mu = \phi^0_0 (p^\mu)_{\text{local}} = \frac{p_f \phi^0_0}{c} \phi^\mu_\nu u^\nu. \quad (6.24)$$

The output axes remain local, while u^ν has coordinate components. In the zero-shift map, $M_r = p_r^0 = \phi^0_0 M = -p_0$. Contracting the reference vector gives

$$(p_r^0)^2 - \delta_{ij} p_r^i p_r^j = - \left(\frac{p_f \phi^0_0}{c} \right)^2 g_{\mu\nu} u^\mu u^\nu = (\phi^0_0 p_f)^2. \quad (6.25)$$

Thus $p_{f,r} = \phi^0_0 p_f$ and $p_r = \sqrt{\delta_{ij} p_r^i p_r^j}$ obey

$$M_r^2 = p_{f,r}^2 + p_r^2. \quad (6.26)$$

For any unit spatial direction k^i in the same output axes, $p_{k,r} = \delta_{ij} k^i p_r^j$. The opposed values follow by projection:

$$p_{k,r}^\pm = M_r \pm \frac{1}{2} p_{k,r} = \frac{p_f \phi^0_0}{c} \left(\phi^0_\nu \pm \frac{1}{2} \delta_{ij} k^i \phi^j_\nu \right) u^\nu. \quad (6.27)$$

In a plane containing the basic direction, $p_{k,r} = p_r \cos \alpha$. Plotting the plus value along each ray therefore gives $R(\alpha) = M_r + (p_r/2) \cos \alpha$; the opposite ray gives the minus value. The core and fermic reference circles have radii M_r and $p_{f,r}$, respectively.

For light, use $p_r^\mu = \phi^0_0 (p^\mu)_{\text{local}}$ with nonzero null momentum. Its norm vanishes and $p_r = M_r > 0$; the proper-time formula above requires $p_f > 0$.

Stationary core and contextual components

Along a trajectory, the total derivative of the positive root equals its explicit reference-time derivative:

$$\frac{dM_r}{dx^0} = \frac{d(-p_0)}{dx^0} = M \partial_0 \phi^0_0 + \frac{\phi^0_0}{2M} (\partial_0 g^{ij}) p_i p_j. \quad (6.28)$$

It vanishes for a stationary map. At fixed M_r , the squared fermic and basic changes in Equation 6.26 cancel. Local $M = M_r / \phi^0_0$ need not be constant along the trajectory. The local shell retains p_f as its coefficient, while the reference shell uses $p_{f,r}$.

The momenta conjugate to the coordinates x^i are p_i . Their canonical one-form uses the spatial map,

$$p_i dx^i = p_j^{\text{local}} \phi^j_i dx^i,$$

The reference triangle instead uses $\phi^0_0 \mathbf{p}_{\text{local}}$. Substituting those components into the action without transforming the coordinates and symplectic structure would change the dynamics.

Reference normalization and physical internal length

The common factor is a declared comparison. If the reference observer has temporal coefficient N_o , replace $N = \phi^0_0$ in the normalized readings by N/N_o ; the chapter chooses $N_o = 1$. A constant change $\tilde{x}^0 = ax^0$ sends both N and N_o to their values divided by a . The normalized readings are unchanged, whereas the coordinate generator $-p_0$ scales as $1/a$. Its conservation statement always refers to the chosen stationary time.

The reference triangle does not uniquely determine a physical internal radius. For a selected cyclic pair (ϑ, I) with $I > 0$, $R_0 > 0$ and $p_f = I/R_0$, compare the two extended line elements

$$d\sigma_1^2 = -(dx^0)^2 + N^{-2} g_{ij} dx^i dx^j + (R_0/N)^2 d\vartheta^2, \quad d\sigma_2^2 = N^2 d\sigma_1^2.$$

Their null constraints differ by the positive factor N^{-2} and give the same root $-p_0 = N\sqrt{I^2/R_0^2 + g^{ij}p_i p_j}$ after rescaling the constraint multiplier. They have the same reduced trajectories and selected cycle phase, although the displayed circle radii are R_0/N and R_0 . These comparison observables alone therefore do not select which internal length is physical. A separate internal constitutive law must make that assignment.

In particular, replacing the **local** shell coefficient by Np_f while retaining the same temporal conversion gives the different root $N\sqrt{(Np_f)^2 + |\mathbf{p}_{\text{local}}|^2}$. Its held value is $N^2 p_f$, not Np_f . The intended no-exchange redistribution requires a consistently derived internal response and source, not a second use of one reference factor.

Axial motion and the shell figures

For motion along $+x$ with a diagonal spatial map, $p_1 = \phi^1_1 p_{\text{local}}^1$ and $g^{11} = 1/(\phi^1_1)^2$. Substitution into Equation 6.22 gives

$$\frac{v}{c} = \frac{dx^1}{dx^0} = \frac{\phi^0_0}{\phi^1_1} \frac{p_{\text{local}}^1}{M} = \frac{\phi^0_0}{\phi^1_1} \frac{p_r}{M_r}. \quad (6.29)$$

At the same reference velocity, the basic-to-core ratio depends on the local map. The shell comparison in §3.5 holds the reference velocity fixed and calculates the two basic momenta separately.

For release from rest in the distant limit, $\phi^0_0 \rightarrow 1$ fixes $M_r = p_f$. The reference triangle then gives

$$p_r^2 = p_f^2 - p_{f,r}^2 = p_f^2 (1 - [\phi^0_0]^2). \quad (6.30)$$

This gives the free-fall sequence in §3.5. A later interaction that stops the carrier at a fixed depth leaves $p_r = 0$ and $M_r = p_{f,r}$, transferring $p_f - p_{f,r}$ out of the carrier. The endpoint balance fixes this amount; the stopping interaction must be supplied separately.

The specified moving and held cycle

For the same massive carrier, the constrained part of the action vanishes on the shell. Using $c d\tau/d\lambda = ep_f$, the action along the trajectory becomes

$$dS = p_\mu dx^\mu = -cp_f d\tau.$$

The chosen action phase consequently satisfies

$$d\varphi = \frac{cp_f}{\hbar} d\tau. \quad (6.31)$$

In reference coordinates, $\dot{x}^0 = eM/\phi^0_0$. Hence

$$\frac{d\tau}{dt} = \phi^0_0 \frac{p_f}{M}, \quad \frac{d\varphi}{dt} = \frac{cp_f}{\hbar} \phi^0_0 \frac{p_f}{M}. \quad (6.32)$$

Equivalently, $d\varphi/dt = (c/\hbar)p_{f,r}^2/(-p_0) = (c/\hbar)p_{f,r}^2/M_r$. The same result follows directly from the reduced action: $p_i(dx^i/dx^0) - (-p_0) = -\phi^0_0 p_f^2/M$. In flat space, $\phi^0_0 = 1$, and the cycle-rate factor reduces to the Foundations ratio p_f/M .

For a held ideal carrier, $\mathbf{p}_{\text{local}} = 0$ and $M = p_f$ in this free kinetic description. If the support does not perturb the chosen cycle,

$$\left(\frac{d\varphi}{dt}\right)_{\text{held}} = \frac{cp_f}{\hbar} \phi^0_0 = \frac{cp_{f,r}}{\hbar}. \quad (6.33)$$

Applying this ideal-cycle rate to a material clock requires its internal motion, stresses and response to the support to justify the approximation.

Result

The free-carrier action supplies §3.5's motion equations and its four-momentum-to-shell construction. In a prescribed zero-shift stationary map it conserves $M_r = -p_0 = \phi^0_0 M$, while the reference triangle exhibits fermic-to-bosic redistribution at fixed reference core. The additional action-phase hypothesis supplies the stated moving and held ideal-clock rates.

Notes

The shell represents the particle's momentum state; a microscopic density or spatial boundary needs further structure. In the free-carrier calculation the bosic momentum describes translation. Internal bosic compensation, source stresses and physical recurrence in a structured particle require its constrained internal dynamics. Derivation 3.10A develops the operational comparison for the cycle hypothesis used here.

Derivation 3.6A — Reciprocal Sources and Static Feedback

Purpose

The particle momenta found in §3.5 supply the source in §3.6. This derivation obtains their local source contributions and the exact static temporal weighting. It then solves a supported continuum example and compares two scalar models of source feedback.

Starting assumptions

Use signature $(-+++)$, $x^0 = ct$ and the current all-upper momentum source. In the static zero-shift patch, write $N = \phi^0_0 > 0$ and $h_{ij} = g_{ij}$ as local calculation aliases. The retained equation is $G^{\mu\nu} = (8\pi G_N/c^3)J^{\mu\nu}$ in matching frames. Its source includes the complete material stress for the model being solved.

The free-carrier example uses $p_r^\mu = Np_{\text{local}}^\mu$, $M_r = NM$ and local physical-volume population weights. The prescribed scalar control later uses Euclidean volume instead; it is a different field-law problem. Neither the free-particle kernel nor the constant-density continuum resolves a singular particle self-field or supplies a microscopic internal-support law.

Derivation

From particle momentum to the source

For each massive carrier, the normalized four-velocity of §3.5 gives

$$p_{\text{local},a}^\mu = \frac{p_{f,a}}{c} \phi^\mu_\alpha u_a^\alpha, \quad p_{r,a}^\mu = Np_{\text{local},a}^\mu.$$

Consequently

$$\begin{aligned}
(j_a^{\mu\nu})_{\text{local}} &= \frac{p_{\text{local},a}^\mu p_{\text{local},a}^\nu}{M_a} \\
&= \frac{p_{r,a}^\mu p_{r,a}^\nu}{NM_{r,a}} \\
&= \frac{p_{f,a}^2}{c^2 M_a} \phi^\mu{}_\alpha \phi^\nu{}_\beta u_a^\alpha u_a^\beta.
\end{aligned} \tag{6.34}$$

The first two forms also apply to nonzero null momenta; the massive proper-time parametrization in the last form does not. Form each product before taking the physical-volume moment. The blocks are

$$\mathcal{M} = \left\langle \sum_a \frac{M_{r,a}}{N} \right\rangle, \quad \mathcal{P}^i = \left\langle \sum_a \frac{p_{r,a}^i}{N} \right\rangle, \quad \mathcal{C}^{ij} = \left\langle \sum_a \frac{p_{r,a}^i p_{r,a}^j}{NM_{r,a}} \right\rangle. \tag{6.35}$$

These are local source components expressed through reference-normalized inputs. Coordinate components require the separate congruence $J_{\text{coord}} = \phi^{-1} J_{\text{local}} \phi^{-\text{T}}$. Neither operation changes which physical content belongs in J .

The static temporal equation

For

$$ds^2 = -N^2(dx^0)^2 + h_{ij}dx^i dx^j,$$

static curvature gives

$$R_{00} = ND_i D^i N, \quad D_i D^i N = \frac{1}{\sqrt{\det h}} \partial_i \left(\sqrt{\det h} h^{ij} \partial_j N \right).$$

Trace reversal of the retained equation, in the local orthonormal frame, gives

$$R_{\text{local},00} = \frac{4\pi G_N}{c^3} (\mathcal{M} + \delta_{ij} \mathcal{C}^{ij}).$$

Since $R_{\text{local},00} = R_{00}/N^2$, the exact result is

$$D_i D^i N = \frac{4\pi G_N}{c^3} N (\mathcal{M} + \delta_{ij} \mathcal{C}^{ij}). \tag{6.36}$$

Trace reversal gives the positive spatial-trace term. This equation sources the temporal map N ; weak θ^{00} alone is sourced by $\mathcal{M}$.

For resting free carriers, $\mathcal{M} = np_f$ and $\mathcal{C}^{ij} = 0$, hence $N\mathcal{M} = np_{f,r}$. The single factor of N converts the local fermic contribution to its reference-expressed value.

The same source must satisfy conservation. For static dust its spatial equation is

$$\mathcal{M} \partial_i \ln N = 0. \quad (6.37)$$

For positive density, this condition requires a spatially constant N . A static body in a varying well therefore needs a supporting stress or force.

A complete static continuum example

Consider an incompressible continuum with constant total local core density, $\mathcal{M} = \mathcal{M}_0$, and isotropic total spatial stress, $\mathcal{C}^{ij} = \mathcal{C}_s(r)\delta^{ij}$. The supporting stress is included in $\mathcal{C}_s$. Let r be areal radius and R_s the surface radius. Define

$$r_s = \frac{8\pi G_N \mathcal{M}_0 R_s^3}{3c^3}, \quad F(r) = 1 - \frac{r_s r^2}{R_s^3}, \quad F_s = 1 - \frac{r_s}{R_s}.$$

Inside the body,

$$N = \frac{3\sqrt{F_s} - \sqrt{F}}{2}, \quad \mathcal{C}_s = \mathcal{M}_0 \frac{\sqrt{F} - \sqrt{F_s}}{3\sqrt{F_s} - \sqrt{F}}, \quad (6.38)$$

and in areal Cartesian coordinates the spatial map is

$$\phi^i_j = \delta^i_j + (F^{-1/2} - 1) \hat{r}^i \hat{r}_j.$$

The centre is defined by smooth extension. These expressions give $ds^2 = -N^2(dx^0)^2 + dr^2/F + r^2 d\Omega^2$. Outside, $F = 1 - r_s/r$, $N = \sqrt{F}$ and $\mathcal{C}_s = 0$.

The radial field and material equations can be written using the enclosed core parameter M_{enc} :

$$\begin{aligned}
\frac{dM_{\text{enc}}}{dr} &= 4\pi r^2 \mathcal{M}_0, \\
\frac{N'}{N} &= \frac{G_N}{c^3} \frac{M_{\text{enc}} + 4\pi r^3 \mathcal{C}_s}{r^2 [1 - 2G_N M_{\text{enc}} / (c^3 r)]}, \\
\mathcal{C}'_s &= -(\mathcal{M}_0 + \mathcal{C}_s) \frac{N'}{N}.
\end{aligned} \tag{6.39}$$

The displayed solution satisfies these equations and the tangential field equation. At R_s , its lapse and radial geometry match the vacuum exterior and its stress vanishes, so no surface stress layer is required. Positive central lapse and finite central stress require $0 < r_s/R_s < 8/9$.

Temporal weighting and the exterior source

For a regular isolated static spherical solution, integrating the temporal equation gives the asymptotic core parameter, in momentum units,

$$M_\infty = \frac{c^3}{G_N} \lim_{r \rightarrow \infty} r^2 \sqrt{F} N' = \int N (\mathcal{M} + \delta_{ij} \mathcal{C}^{ij}) d\Sigma. \tag{6.40}$$

For the supported uniform sphere, $d\Sigma = 4\pi r^2 dr / \sqrt{F}$, and direct substitution gives

$$\frac{N(\mathcal{M}_0 + 3\mathcal{C}_s)}{\sqrt{F}} = \mathcal{M}_0.$$

Thus $M_\infty = 4\pi \mathcal{M}_0 R_s^3 / 3$. The proper integral of local cores alone is larger:

$$\int \mathcal{M}_0 d\Sigma = 4\pi \mathcal{M}_0 \int_0^{R_s} \frac{r^2}{\sqrt{F}} dr > M_\infty.$$

The exterior source depends on temporal weighting, supporting stress and physical volume together.

A scalar feedback model

Now prescribe constant n per **Euclidean** volume, retain only θ^{00} , and add the field-law postulate

$$\Delta_{\text{flat}}\theta^{00} = -\frac{4\pi G_N n p_f}{c^3} \sqrt{1 - 2\theta^{00}} \quad (r < R_s).$$

Require a regular centre, decay at infinity and continuous value and first radial derivative at the surface. With

$$x = \frac{r}{R_s}, \quad \alpha = \frac{4\pi G_N n p_f R_s^2}{c^3}, \quad y = 1 - 2\theta^{00} = N^2,$$

the boundary-value problem becomes

$$y'' + \frac{2}{x}y' = 2\alpha\sqrt{y}, \quad y'(0) = 0, \quad y(1) + y'(1) = 1. \quad (6.41)$$

Primes now mean d/dx . Outside, $y = 1 - b/x$ with $b = y'(1)$. The reduced-to-unreduced source fraction is

$$S = 3 \int_0^1 x^2 \sqrt{y} dx, \quad b = \frac{2\alpha}{3} S.$$

Solving without expanding the square root gives:

α	Central N	Surface N	Source fraction S
0.1	0.950833949	0.967443049	0.960809196
1	0.581376331	0.740308770	0.677914387
2	0.302614700	0.596052040	0.483541474
4	0.019726127	0.465118871	0.293874163

The tabulated values are numerical solutions of this scalar boundary-value problem, checked by shooting, collocation and the source–flux identity. They are not solutions of all tensor field equations. The flat operator and Euclidean density are constitutive choices; a physical spatial metric and reciprocal material dynamics cannot be inferred from this scalar equation alone.

Giulini's homogeneous scalar sphere

Giulini's account of Einstein's Prague field equation, §§2–4, supplies a related exact scalar example. For a uniform bare mass m_b inside R_s , his normalized scalar $\Psi = \sqrt{\Phi/c^2}$ satisfies

$$\Delta_{\text{flat}} \Psi = \kappa^2 \Psi, \quad \kappa^2 = \frac{3G_N m_b}{2c^2 R_s^3}, \quad \Psi(\infty) = 1.$$

Put $z = \kappa R_s$. Regularity and matching give

$$\Psi_{\text{in}}(r) = \frac{\sinh(\kappa r)}{\kappa r \cosh z}, \quad \Psi_{\text{out}}(r) = 1 - \frac{R_g}{r},$$

$$R_g = R_s \left(1 - \frac{\tanh z}{z}\right) = \frac{G_N m_g}{2c^2}, \quad \frac{m_g}{m_b} = \frac{3}{z^2} \left(1 - \frac{\tanh z}{z}\right). \quad (6.42)$$

The active mass m_g includes the scalar field's contribution in that theory. At $z = 1$, $m_g/m_b = 0.715217532$; its matter and field energy fractions are respectively 0.512429722 and 0.202787811.

Identifying the bare mass density with np_f/c gives $z^2 = \alpha/2$ in Giulini's model. A flat-spatial version of the retained resting-lapse equation would instead have $z^2 = \alpha$, while the nonlinear- θ problem above has a different field equation again. These comparisons relate their source parameters; each model retains its own force law and source accounting.

Result

The reference shell enters the kinetic source through $j_a^{\mu\nu} = p_{r,a}^\mu p_{r,a}^\nu / (NM_{r,a})$. The exact static temporal equation weights a resting contribution as $np_{f,r}$. The supported continuum solution shows how pressure and curved volume enter the complete source, while the two scalar examples describe feedback under their own field-law postulates.

Notes

The supported uniform sphere is the interior Schwarzschild incompressible continuum in momentum units. Its stress is specified at the continuum level; internal particle support and causal material evolution require further physical laws. Giulini's formulas used here are his equations (24), (32)–(33) and (37). The calculations support §3.6.

Derivation 3.7A — Field Equations and Momentum Balance

Purpose

The finite field law of §3.7 becomes a system of equations for the registered deformation. This derivation writes out that system, obtains its weak wave equation and initial constraints, and derives the local and finite-region momentum balances. A final common-frame argument gives conserved opposed directional amounts.

Starting assumptions

Use the torsion-free metric connection, signature $(-+++)$ and $x^0 = ct$. The registered reference form η is fixed. Work where the spatial metric is positive definite and the future temporal map factor is positive. The selected matter law supplies the complete source, including any binding and support stresses.

The Einstein field action with zero cosmological term is retained. Take compactly supported variations so that gravitational boundary terms do not enter the local Euler equation.

Derivation

The exact operator in deformation variables

The finite dictionary is

$$g_{\mu\nu}(\theta) = \eta_{\mu\nu} + 4\eta_{\mu\alpha}\theta^{\alpha\beta}\eta_{\beta\nu} - 2\eta_{\mu\nu}\eta_{\alpha\beta}\theta^{\alpha\beta}. \quad (6.43)$$

Form $g^{\mu\nu}$ by inverting this finite matrix. Its inverse and derivatives determine the connection and curvature:

$$\Gamma^\alpha{}_{\mu\nu} = \frac{1}{2}g^{\alpha\beta}(\partial_\mu g_{\beta\nu} + \partial_\nu g_{\beta\mu} - \partial_\beta g_{\mu\nu}),$$

$$R_{\mu\nu} = \partial_\alpha \Gamma^\alpha_{\mu\nu} - \partial_\nu \Gamma^\alpha_{\mu\alpha} + \Gamma^\alpha_{\alpha\beta} \Gamma^\beta_{\mu\nu} - \Gamma^\alpha_{\nu\beta} \Gamma^\beta_{\mu\alpha},$$

$$R = g^{\mu\nu} R_{\mu\nu}, \quad G^{\mu\nu} = g^{\mu\alpha} g^{\nu\beta} R_{\alpha\beta} - \frac{1}{2} g^{\mu\nu} R. \quad (6.44)$$

Together these formulas give ten nonlinear second-order field equations for θ . Although $g_{\mu\nu}$ is affine in θ , its inverse, the connection products and the curvature contractions depend nonlinearly on the finite deformation.

Action normalization and full-rank variation

The retained gravitational action and matter variation are

$$S_g = \frac{c^3}{16\pi G_N} \int \sqrt{-g} R d^4x, \quad \delta S_m = \frac{1}{2} \int \sqrt{-g} J_{\text{coord}}^{\mu\nu} \delta g_{\mu\nu} d^4x.$$

Stationarity gives

$$G^{\mu\nu} = \frac{8\pi G_N}{c^3} J_{\text{coord}}^{\mu\nu}, \quad J_{\text{coord}} = \phi^{-1} J_{\text{local}} \phi^{-\text{T}}. \quad (6.45)$$

Let $E^{\mu\nu}$ be the symmetric residual multiplying $\delta g_{\mu\nu}$. Varying θ directly gives the coefficient of unrestricted symmetric $\delta\theta^{\alpha\beta}$:

$$4\eta_{\alpha\mu} E^{\mu\nu} \eta_{\nu\beta} - 2\eta_{\alpha\beta} \eta_{\mu\nu} E^{\mu\nu}.$$

Contracting with $\eta^{\alpha\beta}$ first makes the reference trace of E vanish; the remaining equation then gives $E = 0$. Thus the ten deformation equations and ten metric equations have exactly the same zero set on the regular chart. The positive-root choice for the spatial coframe fixes its orientation, not the allowed metric variations.

Weak propagation and the stationary limit

Write $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ for this linearization only, and define its trace reverse $\bar{h}_{\mu\nu} = h_{\mu\nu} - \eta_{\mu\nu}\eta^{\alpha\beta}h_{\alpha\beta}/2$. The dictionary gives

$$\bar{h}^{\mu\nu} = 4\theta^{\mu\nu}.$$

In aligned harmonic coordinates, $\partial_\mu\theta^{\mu\nu} = 0$, so

$$G^{(1)\mu\nu} = -2\Box\theta^{\mu\nu}, \quad \Box\theta^{\mu\nu} = -\frac{4\pi G_N}{c^3}J^{\mu\nu}. \quad (6.46)$$

At this order local and coordinate leading sources coincide in the reference frame. A stationary conserved localized source, with response vanishing at infinity, gives

$$\theta^{\mu\nu}(\mathbf{x}) = \frac{G_N}{c^3} \int \frac{J^{\mu\nu}(\mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|} d^3x'.$$

The integral solves the stationary weak equation. At finite deformation, the full operator also contains the inverse-metric and curvature-product terms above.

Initial constraints and local evolution

Let h_{ij} now denote the induced spatial metric, n^μ the future unit normal and $K_{ij} = -\frac{1}{2}\mathcal{L}_n h_{ij}$ the extrinsic curvature. Use temporary normal-frame source projections

$$\mathcal{M}_n = J_{\mu\nu}n^\mu n^\nu, \quad \mathcal{P}_{n,i} = -J_{i\nu}n^\nu,$$

where the coordinate source has been lowered with g . The four initial constraints are

$${}^{(3)}R + K^2 - K_{ij}K^{ij} = \frac{16\pi G_N}{c^3}\mathcal{M}_n, \quad D_j K^j_i - D_i K = \frac{8\pi G_N}{c^3}\mathcal{P}_{n,i}. \quad (6.47)$$

The six spatial equations evolve the spatial geometry with the matter. In a suitable wave-coordinate formulation, the coordinate-condition residual obeys a homogeneous wave equation when the matter balance holds. Compatible initial gauge and constraint data therefore remain compatible in their domain of dependence. The metric principal symbol is $g^{\alpha\beta}\xi_\alpha\xi_\beta$: the gravitational characteristics are null, and the usual constraint/gauge count leaves two local radiative polarizations.

For the local existence statement, take smooth constraint-satisfying data on a spacelike Cauchy slice, a smooth nonnegative distribution with compact momentum support, and a regular chart

whose spatial metric is positive definite and whose lapse stays positive. Work in a boundary-free neighborhood or its domain of dependence. For positive-mass species the local result follows from Choquet-Bruhat's Einstein–Liouville Cauchy theorem, §10; smooth compact-support data form a sufficient subclass of its weighted Sobolev hypotheses. For a purely massless population use the separately stated local Cauchy result, Theorem 12.1 of Taylor, with momentum support restricted to a compact subset of the nonzero future null shell.

The coframe and canonical/local momentum maps are smooth and invertible on compact regular patches; support away from zero bounds the null inverse-core weights. The free source and transport equations are those of these comparison systems. Their local geometric existence and uniqueness therefore transfer to the registered variables while the chart remains regular. This imports the stated local results, not a new PDE theorem, a proof of strong hyperbolicity for arbitrary raw evolution variables, an arbitrary mixed-shell or timelike-boundary theorem, or global regularity. The explicit mixed homogeneous example in Derivation 3.8A is a separate construction.

Local and finite-window momentum balance

The matter equations must supply $\nabla_\mu J^{\mu\nu} = 0$, consistently with the contracted curvature identity. In all-upper coordinate components,

$$\partial_\mu (\sqrt{-g} J^{\mu\nu}) = -\sqrt{-g} \Gamma^\nu_{\mu\alpha} J^{\mu\alpha}. \quad (6.48)$$

For an explicitly chosen covector K_ν that specifies a component comparison between events,

$$\partial_\mu (\sqrt{-g} J^{\mu\nu} K_\nu) = \sqrt{-g} J^{\mu\nu} \nabla_\mu K_\nu.$$

Integrate against a smooth compact spatial window $w(x^0, \mathbf{x})$ and integrate the spatial divergence by parts:

$$\begin{aligned} \frac{d}{dx^0} \int w \sqrt{-g} J^{0\nu} K_\nu d^3x &= \int \sqrt{-g} [w J^{\mu\nu} \nabla_\mu K_\nu \\ &\quad + (\partial_0 w) J^{0\nu} K_\nu + (\partial_i w) J^{i\nu} K_\nu] d^3x. \end{aligned} \quad (6.49)$$

The right-hand side separates changing component comparison, window motion and boundary transport. An average per physical volume also acquires a term from that volume's variation. Its evolution therefore requires these flux, comparison and normalization terms as well as the averaged tensor. For a sharply bounded moving region, the surface flux is measured relative to the moving boundary.

Directional conservation in a common frame

In a common flat frame, suppose the **complete** account satisfies

$$\partial_0 \mathcal{M} + \partial_i \mathcal{P}^i = 0, \quad \partial_0 \mathcal{P}^i + \partial_j \mathcal{C}^{ji} = 0, \quad \mathcal{M} \geq |\mathcal{P}|.$$

For a fixed unit direction $\hat{k}$, linear combination gives

$$\partial_0 \mathcal{J}_k^\pm + \partial_j \left(\mathcal{P}^j \pm \frac{1}{2} \hat{k}_i \mathcal{C}^{ji} \right) = 0. \quad (6.50)$$

If the corresponding boundary flux vanishes, each integrated opposed amount is conserved. Its density obeys $\mathcal{J}_k^\pm \geq \mathcal{M}/2$, so it is positive for nonzero total core. For the free kinetic case, the triangle inequality and $|\mathbf{p}_a| \leq M_a$ prove the assumed bound under nonnegative weights.

Momentum transmitted through a supporting wall belongs in this balance. In curved space, extending the argument requires a common-direction comparison, the field contribution and a positive microscopic allocation. Covariant conservation supplies the local balance but not those additional ingredients.

Result

The retained finite operator preserves all Einstein equations in the registered deformation. Its weak harmonic reduction is the sourced wave equation, and its stationary restriction gives the source integral. Local conservation supplies exact finite-region balances; positive conserved opposed totals follow under the additional common-frame, closure and positivity assumptions stated above.

Notes

The field action remains a premise. A different field or matter action requires a fresh derivation of its source, constraints, characteristics and conservation laws. Within the stated premises, the results support §3.7.

Derivation 3.8A — Time-Dependent Transport and Coupled Sources

Purpose

Free carriers move through a changing geometry while contributing to its source. This derivation obtains their transport law, the causal weak-field response and an exact homogeneous source–field solution. A separate effective material law gives the finite-amplitude body example used in §3.8.

Starting assumptions

Retain the field and free-carrier coupling of Derivations 3.4A and 3.7A. Coordinate momenta are raised with g ; local momenta use the orthonormal frame. The future shell has $p^0 > 0$ in the admissible time slicing. Nonzero null carriers are permitted, but their zero-momentum vertex is excluded. The material evolution described later uses a different, explicitly adopted effective matter law, not a sum of unsupported free dyads.

Derivation

Transport without stationarity

Variation of the free constrained carrier action gives, for a parameter λ and multiplier e ,

$$\frac{dx^\mu}{d\lambda} = eg^{\mu\nu}p_\nu, \quad \frac{dp_\mu}{d\lambda} = -\frac{e}{2}\partial_\mu g^{\alpha\beta}p_\alpha p_\beta, \quad g^{\mu\nu}p_\mu p_\nu = -p_f^2.$$

Dividing by $dx^0/d\lambda = ep^0$ gives §3.8's transport equations and

$$\frac{d(-p_0)}{dx^0} = \frac{1}{2p^0}\partial_0 g^{\alpha\beta}p_\alpha p_\beta. \tag{6.51}$$

Stationarity conserves $-p_0$ even with shift; zero shift additionally gives $-p_0 = NM$. Here M is the local core, while p^0 and p_0 are the coordinate contravariant and covariant temporal components.

For a distribution f on canonical spatial phase space, conservation along this Hamiltonian flow is

$$\partial_0 f + \partial_i \left(f \frac{p^i}{p^0} \right) + \partial_{p_i} \left(f \frac{dp_i}{dx^0} \right) = 0.$$

The phase and future-shell measures satisfy

$$d^3x d^3p_{\text{coord}} = d\Sigma d^3p_{\text{local}}, \quad \frac{d^3p_{\text{coord}}}{\sqrt{-g}p^0} = \frac{d^3p_{\text{local}}}{M}. \quad (6.52)$$

Taking moments of transport with decaying or compact momentum support gives $\nabla_\mu J^{\mu\nu} = 0$. Thus the matter equations and invariant measure supply the source balance required by the field equation.

Retarded fields and two tidal polarizations

The retarded Green function for the weak harmonic equation yields

$$\theta^{\mu\nu}(t, \mathbf{x}) = \frac{G_N}{c^3} \int \frac{J^{\mu\nu}(t - R/c, \mathbf{x}')}{R} d^3x', \quad R = |\mathbf{x} - \mathbf{x}'|.$$

For a conserved leading source and a convergent convolution, retarded homogeneous data select this sourced response. General initial data can also include free radiation.

For a linear vacuum example, put $u = x^0 - x^3$ and choose $H_{11}(u) = -H_{22}(u)$, $H_{12}(u) = H_{21}(u)$, with all longitudinal entries zero. Then

$$\theta^{ij} = \frac{1}{4} H_{ij}(u), \quad \theta^{00} = \theta^{0i} = 0, \quad R_{0i0j}^{(1)} = -\frac{1}{2} \frac{d^2 H_{ij}}{du^2}. \quad (6.53)$$

The two independent functions solve the linear vacuum equation and constraints and produce nonzero tidal curvature. They describe freely propagating weak waves.

An exact nonlinear homogeneous kinetic solution

Use a flat periodic spatial domain or a local homogeneous patch, and

$$g_{00} = -1, \quad g_{0i} = 0, \quad g_{ij} = a(x^0)^2 \delta_{ij}, \quad a > 0.$$

Let $p_c = |\mathbf{p}_{\text{coord}}|$ be the Euclidean magnitude of the canonical spatial covector in these homogeneous coordinates. For each species s , choose a smooth nonnegative compact radial profile $f_s(p_c)$; null profiles avoid $p_c = 0$. Translation symmetry conserves p_i , while local momentum is p_i/a . Thus

$$\begin{aligned} M_s(a, p_c) &= \sqrt{p_{f,s}^2 + p_c^2/a^2}, \\ \mathcal{M}(a) &= 4\pi a^{-3} \sum_s \int_0^\infty f_s(p_c) M_s p_c^2 dp_c, \\ \mathcal{C}_{\text{iso}}(a) &= \frac{4\pi}{3} a^{-5} \sum_s \int_0^\infty \frac{f_s(p_c) p_c^4}{M_s} dp_c, \quad \mathcal{C}^{ij} = \mathcal{C}_{\text{iso}} \delta^{ij}. \end{aligned} \tag{6.54}$$

Primes in this subsection mean d/dx^0 , and $H_a = a'/a$. Differentiation of the moment integral gives

$$\mathcal{M}' = -3H_a(\mathcal{M} + \mathcal{C}_{\text{iso}}).$$

The nonzero field equations reduce to

$$3H_a^2 = \frac{8\pi G_N}{c^3} \mathcal{M}, \quad H'_a = -\frac{4\pi G_N}{c^3} (\mathcal{M} + \mathcal{C}_{\text{iso}}). \tag{6.55}$$

For a nonzero population the solution is specified by

$$x^0 - x_0^0 = \sigma \int_{a_0}^a \frac{d\alpha}{\alpha \sqrt{8\pi G_N \mathcal{M}(\alpha)/(3c^3)}}, \quad \sigma = \pm 1.$$

On a bounded positive- a interval the integrand is regular. Differentiating the first field equation and using the moment identity yields the second. Direct substitution also satisfies the spatial field equations; the momentum constraints vanish by homogeneity, and the distribution solves transport exactly. The same homogeneous construction accommodates mixtures of massive and massless species.

For purely null matter with $a(0) = 1$, $\mathcal{M} = \mathcal{M}_0 a^{-4}$ and $\mathcal{C}_{\text{iso}} = \mathcal{M}/3$, so

$$a^2 = 1 + 2\sigma\sqrt{\frac{8\pi G_N \mathcal{M}_0}{3c^3}} x^0.$$

The registered deformation is

$$\theta^{00} = \frac{3(a^2 - 1)}{8}, \quad \theta^{0i} = 0, \quad \theta^{ij} = \frac{1 - a^2}{8} \delta^{ij}. \quad (6.56)$$

They reconstruct $\phi = \text{diag}(1, a, a, a)$ exactly. The sign σ allows expanding and contracting branches.

The finite-amplitude material illustration

The material example uses the self-bound fluid specified in §3.10 and Derivation 3.10A: sound-speed parameter $s = 1/3$, central pressure 0.02 times the surface energy density, and units $G_N = c = 1$ with unit surface energy and constituent densities. Its equilibrium radius is 0.0909395683980, and its fundamental linear period in the asymptotic time normalization is $T_0 = 0.345685420215$.

A fixed-number, constraint-compatible initial velocity kick excites the radial mode. The full nonlinear material and current gravitational fields are evolved, with **zero scalar signal**. The separately reviewed finite-amplitude calculation uses $\epsilon = 0.001$ and 0.005 for $0 \leq t \leq 2.0741125213$, six reference linear periods. The parameter ϵ scales the prepared mode kick; the linear displacement is normalized by $\xi(R_s) = R_s$ per unit amplitude.

For $\epsilon = 0.005$, successive maxima of the evolved surface give:

Interval	Period in asymptotic time
1	0.345751716735
2	0.345742850721
3	0.345730312911
4	0.345715330380
5	0.345699161567

The order-64 to order-96 period differences are no greater than 2.31×10^{-9} . The independently evaluated surface-expansion residual decreases across orders 48, 64 and 96 from 2.71×10^{-5} to 1.08×10^{-5} to 3.88×10^{-6} ; it is finite, not exactly enforced at finite resolution. At order 96, maximum local constituent-label drift is 6.39×10^{-9} and the independent mass-work residual is 3.84×10^{-12} .

The evolved body has cycle-dependent recurrence over the tested interval. This numerical result does not establish an exact periodic orbit or nonlinear stability. The $\epsilon = 0.01$ extension

enters tension and worsens local and boundary diagnostics at higher order; it is outside the verified result.

Accumulated time on the moving surface

For the actual surface trajectory $x_s^\mu(t)$,

$$\Delta\tau_s = \int_{t_1}^{t_2} \sqrt{-g_{\mu\nu}(x_s(t))} \frac{dx_s^\mu}{c dt} \frac{dx_s^\nu}{c dt} dt.$$

Independent integration on the evolved body gives successive proper intervals ranging from 0.333259916533 to 0.333209250486 in the same units. A fixed initial static multiplier differs by about 2.01×10^{-5} relatively. Both motion and the evolving geometry must stay in the time integral.

Result

The free coupling supplies time-dependent transport and its source balance. The retained weak field has retarded propagation and two tidal polarizations. A homogeneous distribution supplies an exact finite coupled example, while a separately adopted material law supplies a bounded nonlinear body evolution with measured changing recurrence.

Notes

The homogeneous kinetic solution and the material body use different source laws and boundary conditions. The body evolution has zero scalar signal; the weak outgoing-signal comparison and homogeneous finite-exchange solution in §3.10 are separate calculations. These results support §3.8.

Derivation 3.9A — Four- and Six-Component Forms

Purpose

This derivation proves the direct transformations used in §3.9. It distinguishes a six-component carrier vector from six aggregate first moments and from the full correlation tensor, and constructs the metric, map and field equation on the physical four-dimensional support.

Starting assumptions

Order the component labels as $(1+, 1-, 2+, 2-, 3+, 3-)$. Write $A = (i, \sigma)$, with fixed axis label i and branch $\sigma = \pm 1$. The four-component local vector is $(M, p_1, p_2, p_3)^\top$, with signed spatial momenta. The Foundations normalization is $p^{i\sigma} = M + \sigma p_i/2$.

The primary source J and deformation θ are all-upper arrays. The reference form is $\eta_{(4)} = \text{diag}(-1, 1, 1, 1)$. Representation subscripts distinguish the four- and six-component forms.

Derivation

Lift, inverse and support

The forward and inverse maps both use the symbol L ; the indices specify the direction:

$$[L^A_\mu] = \begin{pmatrix} 1 & 1/2 & 0 & 0 \\ 1 & -1/2 & 0 & 0 \\ 1 & 0 & 1/2 & 0 \\ 1 & 0 & -1/2 & 0 \\ 1 & 0 & 0 & 1/2 \\ 1 & 0 & 0 & -1/2 \end{pmatrix}, \quad [L^\mu_A] = \begin{pmatrix} 1/6 & 1/6 & 1/6 & 1/6 & 1/6 & 1/6 \\ 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \end{pmatrix}. \quad (6.57)$$

They satisfy

$$L^\mu{}_A L^A{}_\nu = \delta^\mu{}_\nu, \quad \Pi^A{}_B = L^A{}_\mu L^\mu{}_B, \quad \Pi^2 = \Pi = \Pi^\top, \quad \text{rank } \Pi = 4.$$

In component form,

$$\Pi^{i\sigma}{}_{j\tau} = \frac{1}{6} + \frac{\sigma\tau}{2}\delta_{ij}.$$

There is no implicit sum over the fixed component labels in this formula. A vector belongs to the image precisely when its three opposed-pair sums agree. Their common half-sum gives M , and their differences give the three p_i . The particle's mass shell imposes a further condition within this image.

Correlations must precede averaging

For a kinetic population, form the products on each carrier:

$$J^{AB} = L^A{}_\mu L^B{}_\nu J^{\mu\nu}, \quad J^{\mu\nu} = L^\mu{}_A L^\nu{}_B J^{AB}.$$

Explicitly,

$$J^{i\sigma,j\tau} = \mathcal{M} + \frac{\sigma}{2}\mathcal{P}^i + \frac{\tau}{2}\mathcal{P}^j + \frac{\sigma\tau}{4}\mathcal{C}^{ij}. \quad (6.58)$$

The first moments are recovered from the row sums:

$$\mathcal{J}^{i\sigma} = \frac{1}{6} \sum_{j,\tau} J^{i\sigma,j\tau} = \mathcal{M} + \frac{\sigma}{2}\mathcal{P}^i.$$

They do not determine $\mathcal{C}^{ij}$. For example, equal streams with momenta $\pm p\hat{x}$ and equal streams with momenta $\pm p\hat{y}$, using the same cores and weights, both have zero net spatial momentum and identical first moments. The former has transport along x , the latter along y . Multiplying their aggregate component values would erase this difference.

For two equal carriers with local core M and momenta $+p\hat{x}$ and $-p\hat{x}$, the relevant four directional readings are

$$p_\pm^{1+} = M \pm \frac{p}{2}, \quad p_\pm^{1-} = M \mp \frac{p}{2}, \quad p_\pm^{2+} = p_\pm^{2-} = M.$$

Therefore, for unit carrier weights,

$$J_x^{1+,1+} - J_x^{1+,1-} = \sum_{\pm} \frac{(p_{\pm}^{1+})^2 - p_{\pm}^{1+} p_{\pm}^{1-}}{M} = \frac{p^2}{M},$$

while

$$J_x^{2+,2+} - J_x^{2+,2-} = 0.$$

For equal streams along y , the same nonzero difference appears in the 2 pair instead. The first moments remain $\mathcal{J}^{i\sigma} = \mathcal{M}$ in both examples, but the correlation tensor locates the transport direction.

Both the source and the deformation obey the support condition $\Pi X \Pi = X$. This restricts a symmetric six-component tensor to ten independent components, the same as a symmetric four-by-four tensor.

Reference form, metric and operational map

The lower reference form and its supported upper inverse are

$$\eta_{AB} = L^\mu{}_A L^\nu{}_B \eta_{\mu\nu}, \quad \eta^{AB} = L^A{}_\mu L^B{}_\nu \eta^{\mu\nu}.$$

Equivalently, the lower reference form is the shell invariant

$$\eta_{AB} p^A p^B = \sum_{k=1}^3 (p^{k+} - p^{k-})^2 - \left(\frac{1}{6} \sum_A p^A \right)^2 = -p_f^2.$$

This equality uses the local fixed- p_f shell. For the static zero-shift reference readings $p_r^A = N p^A$, it instead becomes

$$\eta_{AB} p_r^A p_r^B = -N^2 p_f^2 = -p_{f,r}^2.$$

Neither equation identifies local momentum with reference-expressed momentum. The local kinetic source written using reference inputs has denominator $N M_r$, not merely M_r :

$$(J^{AB})_{\text{local}} = \left\langle \sum_a \frac{p_{r,a}^A p_{r,a}^B}{N M_{r,a}} \right\rangle.$$

The reference form's entries and supported upper inverse are

$$\eta_{i\sigma,j\tau} = -\frac{1}{36} + \sigma\tau\delta_{ij}, \quad \eta^{i\sigma,j\tau} = -1 + \frac{\sigma\tau}{4}\delta_{ij}.$$

The product is Π , not I_6 . The reference form is Lorentzian on its support and has two redundant null directions off it. Lowered component momenta are dual components, not the positive opposed values.

The current all-upper deformation transforms directly:

$$\theta^{AB} = L^A{}_\mu L^B{}_\nu \theta^{\mu\nu}, \quad \theta^{\mu\nu} = L^\mu{}_A L^\nu{}_B \theta^{AB}.$$

The metric and supported inverse are

$$g_{AB} = L^\mu{}_A L^\nu{}_B g_{\mu\nu}, \quad g^{AB} = L^A{}_\mu L^B{}_\nu g^{\mu\nu}.$$

The covariant metric is equivalently obtained from the readout increments $d\ell^A = \phi^A{}_B dx^B$:

$$g_{AB} dx^A dx^B = \sum_{k=1}^3 (d\ell^{k+} - d\ell^{k-})^2 - \left(\frac{1}{6} \sum_A d\ell^A \right)^2,$$

or componentwise $g_{AB} = \eta_{CD} \phi^C{}_A \phi^D{}_B$. They satisfy the same registered dictionary on support,

$$g_{AB} = \eta_{AB} + 4\eta_{AC} \theta^{CD} \eta_{DB} - 2\eta_{AB} \eta_{CD} \theta^{CD}. \quad (6.59)$$

Recover the four-dimensional metric by $g_{\mu\nu} = L^A{}_\mu L^B{}_\nu g_{AB}$. Its induced spatial metric determines physical volume.

The operational map and supported inverse preserve slot ownership:

$$\phi^A{}_B = L^A{}_\mu \phi^\mu{}_\nu L^\nu{}_B, \quad (\phi^{-1})^A{}_B = L^A{}_\mu (\phi^{-1})^\mu{}_\nu L^\nu{}_B. \quad (6.60)$$

Their products are the appropriate support projectors. In the undeformed reference, $\phi_{(6)} = \Pi$. To construct the map, first reconstruct $\phi_{(4)}$ with the fixed positive spatial square root, then lift it.

Tensor transformations and source weights

For a fixed four-component frame change Λ , the supported vector map is $U^A{}_B = L^A{}_\mu \Lambda^\mu{}_\nu L^\nu{}_B$. A consistently formed all-upper source transforms by congruence,

$$J'_{(6)} = U J_{(6)} U^\top.$$

Its metric-lowered mixed operator transforms by similarity instead. The same distinction applies to field operators. The isolated particle kernel retains its density weight:

$$j'_a = \frac{M_a}{M'_a} \Lambda j_a \Lambda^\top.$$

The distribution or slice weight supplies the complementary factor, so the aggregate source transforms as a tensor. An already formed local tensor converts to coordinate components by $\phi^{-1} J \phi^{-\top}$; physical-volume normalization is a separate operation.

Supported derivatives, curvature and the field solution

In the following finite-coordinate curvature construction, first convert the source by $J_{(4),\text{coord}} = \phi_{(4)}^{-1} J_{(4),\text{local}} \phi_{(4)}^{-\top}$ and define $J_{\text{coord}}^{AB} = L^A{}_\mu L^B{}_\nu J_{\text{coord}}^{\mu\nu}$. The earlier carrier products were local; at finite deformation they cannot be equated directly to a coordinate curvature tensor.

For a fixed registration $X^A = L^A{}_\mu x^\mu$, define

$$\partial_A = L^\mu{}_A \partial_\mu, \quad \partial_{i\sigma} = \frac{1}{6} \partial_0 + \sigma \partial_i.$$

Then $\eta^{AB} \partial_A \partial_B = \square$. The connection calculated with the supported inverse is

$$\Gamma^A{}_{BC} = \frac{1}{2} g^{AD} (\partial_B g_{DC} + \partial_C g_{DB} - \partial_D g_{BC}) = L^A{}_\rho L^\mu{}_B L^\nu{}_C \Gamma^\rho{}_{\mu\nu}.$$

Curvature contractions therefore reproduce the four-dimensional ones, and

$$G^{AB} = L^A{}_\mu L^B{}_\nu \left(R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R \right) = \frac{8\pi G_N}{c^3} J_{\text{coord}}^{AB} \quad (6.61)$$

with coordinate components on both sides. Both upper slots transform by $L^A{}_\mu$, as they do for the source.

In the aligned weak limit the leading coordinate and local source components agree. The weak harmonic equation and stationary solution then become

$$\square\theta^{AB} = -\frac{4\pi G_N}{c^3}J^{AB}, \quad \theta^{AB}(\mathbf{x}) = \frac{G_N}{c^3} \int \frac{J^{AB}(\mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|} d^3x',$$

under the same leading-source, gauge and decay assumptions as in four components. Applying $L^\mu{}_A$ on both upper slots recovers the original equation and solution.

Position-dependent frame registrations also contribute derivatives of the registration and the compatible frame connection. Fixing the coframe's displayed orientation leaves that transport intact. Covariant conservation on the support is the four-dimensional balance in its six-component representation.

Result

The vector, source, deformation, metric and operational map have exact direct and inverse dictionaries on the rank-four support. The six-component description carries the retained finite field equations and their weak solution on the same rank-four component support. A fixed massive future shell has three independent momentum parameters within that support; this is not a count of gravitational modes. The full correlation tensor supplies the transport information that six aggregate first moments leave unresolved.

Notes

The result applies to the all-upper source and deformation with the fixed-orientation map. The microscopic internal source and curved ADMC account enter later physical work; this derivation supplies the representation used in §3.9.

Derivation 3.10A — GR Correspondence and Material Comparisons

Purpose

The retained field and matter laws give the exact GR correspondence used in §3.10. This derivation develops the counted-pulse experiment, weak and finite gravitational comparisons, and a supported material body whose outgoing signal carries its recurrence. A different prediction requires a specified change in the governing laws.

Starting assumptions

The retained field law is the Einstein equation with zero cosmological term, using the torsion-free metric connection. Free carriers are spinless and minimally coupled. Complete material examples use their explicitly stated effective matter law in place of, not in addition to, a free-particle model for the same constituents. Initial data, boundaries and measuring standards are held fixed in each comparison.

Conventional mass, energy and metric variables enter here to express the correspondence with the preceding momentum-based construction.

Derivation

Exact field and particle correspondence

In matching frames,

$$T^{\mu\nu} = cJ^{\mu\nu}, \quad m_0 = \frac{p_f}{c}, \quad N = \phi^0_0. \quad (6.62)$$

The last identification is the lapse of the coframe-adapted slicing. A coordinate-held clock has rate N only in the zero-shift static comparison, or with its motion relative to that frame otherwise accounted for. The free shell is

$$g^{\mu\nu}p_\mu p_\nu = -m_0^2 c^2,$$

and the retained field equation becomes $G^{\mu\nu} = 8\pi G_N T^{\mu\nu}/c^4$. Derivation 3.7A proves that unrestricted variation in the regular θ chart preserves all metric equations. Derivation 3.8A gives the same particle trajectories, invariant shell measure and collisionless transport as GR.

The equivalence includes finite curvature, the four initial constraints, the two local radiative polarizations and null gravitational characteristics. Local existence and uniqueness use the smooth constrained-data, compact momentum-support and regular-chart hypotheses stated with their primary theorem sources in Derivation 3.7A. Massive and purely massless cases are established separately, with null support away from zero momentum. The change of variables does not itself establish global regularity or an arbitrary-boundary theorem.

The counted-pulse comparison

Let $\Gamma_a = d\varphi_a/dt$ and $\Gamma_b = d\varphi_b/dt$ be the two ideal cycle rates in one stationary coordinate time. If the emitter sends one pulse every m cycles, its emission interval is $2\pi m/\Gamma_a$. For fixed endpoints and a stationary path, the travel time is constant, so the arrival separation in that coordinate time is unchanged. The receiver completes $m\Gamma_b/\Gamma_a$ cycles between arrivals. Hence

$$\mathcal{R}_{a \rightarrow b} = \frac{\Gamma_a}{m\Gamma_b}.$$

For identical held clocks with the selected cycle coupling, $\Gamma_a/\Gamma_b = N_a/N_b$, giving

$$m\mathcal{R}_{a \rightarrow b} = \frac{N_a}{N_b}. \quad (6.63)$$

For unequal fixed free-space cycle scales, multiply the right-hand side by $p_{f,a}/p_{f,b}$. These comparisons assume that support leaves the cycles undisturbed. In a changing geometry, varying travel times also affect the arrival intervals.

Shared weak coefficient

Let $\ell_g = G_N m_{\text{src}}/c^2 = G_N M_{\text{src}}/c^3$. The cold weak isotropic exterior has

$$g_{00} \simeq -1 + \frac{2\ell_g}{r}, \quad g_{ij} \simeq \left(1 + \frac{2\ell_g}{r}\right) \delta_{ij}, \quad N \simeq 1 - \frac{\ell_g}{r}.$$

The slow timelike geodesic gives $\mathbf{a} = -c^2\ell_g\hat{\mathbf{r}}/r^2$ at leading order. The counted-pulse ratio gives

$$m\mathcal{R}_{a\rightarrow b} - 1 \simeq -\ell_g \left(\frac{1}{r_a} - \frac{1}{r_b} \right).$$

For null propagation, the reference-coordinate optical factor is

$$\frac{\sqrt{g_{11}}}{N} \simeq 1 + \frac{2\ell_g}{r}$$

along locally chosen isotropic axes. Integrating its transverse gradient along the leading straight ray gives $\delta_{\text{light}} = 4\ell_g/B$ for distant endpoints and impact parameter B . Integrating its excess along a finite leading ray of Euclidean length L gives the one-way coordinate propagation delay

$$\Delta t_{\text{grav}} \simeq \frac{2\ell_g}{c} \log \frac{r_a + r_b + L}{r_a + r_b - L}. \quad (6.64)$$

A physical timing experiment converts this coordinate delay using its endpoint clocks. Acceleration, counted pulses, bending and delay share the same source normalization.

For a bound massive geodesic of the finite Schwarzschild exterior, let $u = 1/r$ and ψ be orbital angle. With the conserved length $\ell_p = p_\psi/p_f$, the radial equation becomes

$$\frac{d^2u}{d\psi^2} + u = \frac{\ell_g}{\ell_p^2} + 3\ell_g u^2.$$

The leading secular correction to a weak ellipse of semimajor axis a_{orb} and eccentricity e is

$$\Delta\psi \simeq \frac{6\pi\ell_g}{a_{\text{orb}}(1-e^2)}. \quad (6.65)$$

Precession uses the retained finite metric; the preceding force, pulse and light formulas use its weak limit.

A horizon-regular finite map

For the vacuum Schwarzschild solution put $u_r = \sqrt{r_s/r}$, with $r_s = 2\ell_g$ and $r > 0$. The ingoing Painlevé–Gullstrand map has

$$\phi^0_0 = 1, \quad \phi^0_i = 0, \quad \phi^i_j = \delta^i_j, \quad \phi^i_0 = u_r \hat{r}^i.$$

The registered deformation is

$$\theta^{00} = \frac{u_r^2}{8}, \quad \theta^{0i} = -\frac{u_r \hat{r}^i}{4}, \quad \theta^{ij} = \frac{u_r^2}{8} \delta^{ij}. \quad (6.66)$$

They reconstruct $ds^2 = -(dx^0)^2 + |d\mathbf{x} + u_r \hat{\mathbf{r}} dx^0|^2$. A direct constraint check uses flat spatial slices and

$$K_{ij} = \frac{u_r}{r} \left(\delta_{ij} - \frac{3}{2} \hat{r}_i \hat{r}_j \right).$$

Its trace square equals $K_{ij}K^{ij}$, the momentum constraints vanish, and the stationary spatial evolution identities hold. Thus all vacuum equations remain satisfied across $r = r_s$. The coordinate radial null speeds are $-u_r \pm 1$ per unit x^0 , while the local null speed is c .

For a coordinate-held exterior clock, $d\tau/dt = \sqrt{1 - u_r^2}$ because it moves relative to the freely falling local frame. Such held worldlines cease to be timelike at r_s , while the map remains regular. The coordinate construction is also discussed by Martel and Poisson.

Why material stress needs a material law

The kinetic source has $n_i \mathcal{C}^{ij} n_j \geq 0$ for every spatial direction. A binding field may carry tension; for example, a local electric field can have stress–energy entries proportional to $(1, -1, 1, 1)$ in its principal frame. Positive free dyads therefore cannot represent every complete material source.

In a flat stationary isolated balance with vanishing outer traction, integration of $\partial_k(x^i T^{kj})$ gives $\int T^{ij} d^3x = 0$. If all spatial stress were a positive free-dyad sum, this would force its occupied spatial momenta to vanish. The argument uses ordinary flat-space divergence; curved self-gravitating kinetic equilibria obey a covariant balance instead.

One self-bound effective material

For the following material comparison use $G_N = c = 1$. Denote energy density by $\varepsilon(n)$ and pressure by P , with constituent density n . Adopt

$$\varepsilon(n) = \frac{\varepsilon_s}{1+s} \left[s + \left(\frac{n}{n_s} \right)^{1+s} \right], \quad P = n\varepsilon_n - \varepsilon = s(\varepsilon - \varepsilon_s), \quad s = \frac{1}{3}. \quad (6.67)$$

The material occupies a compact domain, and the density law applies only inside it. The exterior contains no material. A free surface has $P = 0$ and $n = n_s$. This is the complete effective material source, including the law's constant contribution once. It replaces the free-dyad model for these constituents rather than being added to it. It is also distinct from the incompressible constant-density sphere of Derivation 3.6A.

With $ds^2 = -e^{2\nu}dt^2 + e^{2\lambda}dr^2 + r^2d\Omega^2$ and $e^{-2\lambda} = 1 - 2m/r$, equilibrium satisfies

$$m' = 4\pi r^2 \varepsilon, \quad P' = -(\varepsilon + P) \frac{m + 4\pi r^3 P}{r(r - 2m)}, \quad \nu' = \frac{m + 4\pi r^3 P}{r(r - 2m)}. \quad (6.68)$$

For $\varepsilon_s = n_s = 1$ and central pressure $P_c = 0.02\varepsilon_s$, the worked solution has

$$R_s = 0.0909395683980106, \quad m(R_s) = 0.00322436115453085, \quad \frac{2m(R_s)}{R_s} = 0.0709121719254032.$$

Analytic bounds give a finite simple zero-pressure surface without a horizon for central-pressure ratios 0.01, 0.02, 0.04. The first and second fundamental forms match the Schwarzschild exterior without a shell. A density scale converts these dimensionless radii and frequencies to physical units.

A derived stable radial recurrence

Let $\xi(r)$ be radial displacement and $\zeta = r^2 e^{-\nu} \xi$. Linearizing the same fluid and gravitational equations gives

$$(A\zeta')' + (B + \omega^2 W)\zeta = 0,$$

with local operator coefficients

$$A = \frac{s(\varepsilon + P)e^{\lambda+3\nu}}{r^2}, \quad W = \frac{(\varepsilon + P)e^{3\lambda+\nu}}{r^2},$$

$$B = \frac{(\varepsilon + P)e^{\lambda+3\nu}}{r^2} \left[(\nu')^2 + \frac{4\nu'}{r} - 8\pi e^{2\lambda} P \right]. \quad (6.69)$$

Regularity selects the central branch. The moving free-surface condition is $\Delta P(R_s) = 0$, hence $\zeta'(R_s) = 0$, because $s(\varepsilon + P)$ remains nonzero at the finite-density surface. This is the standard radial operator, also given by Kokkotas and Ruoff, applied to the specified material and its surface condition.

Integration by parts yields

$$\omega^2 = \frac{\int_0^{R_s} (A\zeta'^2 - B\zeta^2) dr}{\int_0^{R_s} W\zeta^2 dr}.$$

To bound every admissible radial mode, bound the full quadratic form. In dimensionless radius $x = \sqrt{4\pi\varepsilon_s} r$, let $y_c = P_c/\varepsilon_s$ and define

$$X_{\max}^2 = 6y_c, \quad d = 1 - 4(1 + 3y_c)y_c, \quad \nu_{\min} = \frac{1}{2} \log d - \frac{1}{4} \log(1 + 4y_c), \quad V = \frac{(1 + 3y_c)/3 + y_c}{d}.$$

Writing the dimensionless operator coefficients as $\tilde{A}/x^2$, $\tilde{B}/x^2$, $\tilde{W}/x^2$, the equilibrium bounds give

$$\tilde{A} \geq A_{\min} = se^{3\nu_{\min}}, \quad \tilde{B} \leq B_{\max} = (1+4y_c)d^{-1/2}(X_{\max}^2 V^2 + 4V), \quad \tilde{W} \leq W_{\max} = (1+4y_c)d^{-3/2}.$$

Weighted Cauchy–Schwarz gives

$$\int_0^{X_s} \frac{\zeta^2}{x^2} dx \leq \frac{X_s^2}{6} \int_0^{X_s} \frac{\zeta'^2}{x^2} dx, \quad X_s^2 \leq X_{\max}^2.$$

When the numerator is positive,

$$\frac{\omega^2}{4\pi\varepsilon_s} \geq \frac{6A_{\min}/X_{\max}^2 - B_{\max}}{W_{\max}}.$$

For the model above, a conservative exact-rational bound gives

$$\frac{\omega^2}{4\pi\varepsilon_s} \geq \frac{1190922209}{123353008} > 9.65458 \quad (6.70)$$

for every radial eigenvalue. Independent shooting and Galerkin calculations give the nodeless fundamental $\omega_\infty \simeq 18.176020566$ and period $T_\infty \simeq 0.3456854202$. Its surface-area event is

$$A_s(t) = 4\pi R_s^2[1 + 2\epsilon \cos(\omega_\infty t)] + O(\epsilon^2).$$

The bound establishes stability against linear radial perturbations, and the fundamental mode supplies a material recurrence.

A specified signal carries the material cadence

Add a scalar probe ψ with density-linear interaction $\kappa n\psi^2/2$. This is an explicitly adopted signal field, not an internal-radius modulus; its coupling belongs to this material–signal comparison. In these units its equation is

$$\square_g \psi = \kappa n \psi.$$

With $u_\mu u^\mu = -1$, the material-plus-interaction and scalar stresses are

$$T_m^{\mu\nu} = (\varepsilon + P + \kappa n \psi^2/2) u^\mu u^\nu + P g^{\mu\nu},$$

$$T_\psi^{\mu\nu} = \nabla^\mu \psi \nabla^\nu \psi - \frac{1}{2} g^{\mu\nu} \nabla_\alpha \psi \nabla^\alpha \psi.$$

Together with constituent conservation, their exchange is

$$\nabla_\mu T_m^{\mu\nu} = -\kappa n \psi \nabla^\nu \psi, \quad \nabla_\mu T_\psi^{\mu\nu} = +\kappa n \psi \nabla^\nu \psi. \quad (6.71)$$

The total stress sources the retained field equation once. In the small-mode, test-signal calculation, density, metric and moving-interface perturbations produce outgoing quadratures

$$A_{\text{out}}(t) = 1 + \epsilon [b_+ e^{-i\omega_\infty t} + b_- e^{i\omega_\infty t}] + O(\epsilon^2).$$

A static-body control removes the sidebands; changing the incoming carrier leaves their offsets from the carrier, $\pm\omega_\infty$, unchanged. A finite-energy Gaussian train with bandwidth $0.04\omega_\infty$ gives a four-period fit residual 0.001565. Calculated whole-train mode disturbance can be bounded within the chosen classical force model. The outgoing scalar field thus carries the body’s cadence; converting it into recorded events requires a detector model.

Finite exchange and the moving boundary

The full smooth spherical equations for this same material and scalar are also specified. In local comoving derivatives D_t, D_s , let R be areal radius, $U = D_t R$, $\Gamma = D_s R$ and $m = R(1 + U^2 - \Gamma^2)/2$. Define

$$\mathcal{F} = -(D_t \psi)(D_s \psi), \quad \varepsilon_{\text{tot}} = \varepsilon + \kappa n \psi^2 / 2 + \frac{1}{2}[(D_t \psi)^2 + (D_s \psi)^2], \quad P_r = P + \frac{1}{2}[(D_t \psi)^2 + (D_s \psi)^2].$$

The mass constraints and work law include

$$D_s m = 4\pi R^2(\varepsilon_{\text{tot}} \Gamma + \mathcal{F} U), \quad D_t m = -4\pi R^2(P_r U + \mathcal{F} \Gamma). \quad (6.72)$$

The material surface has $n = n_s$. It obeys scalar value and normal-flux continuity, no-shell matching and the compatibility condition $(D_s U - 4\pi R \mathcal{F})/\Gamma = -2U/R$. Finite scalar stress changes the surface mass even though the material pressure vanishes, so the exterior must solve a nonvacuum field problem. With zero probe, the exterior is Schwarzschild.

Derivation 3.8A records the actual six-period, zero-probe nonlinear body evolution at $\epsilon = 0.001, 0.005$, including its finite boundary residual and the unverified larger-amplitude extension. The finite-probe bounded-body equations have not yet been evolved through an outgoing readout experiment.

Finite exchange in a homogeneous system

Finite exchange is demonstrated in a different geometry. Put $n = N_0/a^3$, $v = \dot{\psi}$, $\varepsilon_m = \varepsilon + \kappa n \psi^2 / 2$ and $\varepsilon_\psi = v^2 / 2$. The homogeneous equations are

$$\dot{a} = Ha, \quad \dot{\psi} = v, \quad \dot{v} = -3Hv - \kappa n \psi, \quad \dot{H} = -4\pi(\varepsilon_m + P + 2\varepsilon_\psi),$$

$$H^2 = \frac{8\pi}{3}(\varepsilon_m + \varepsilon_\psi),$$

$$\dot{\varepsilon}_m + 3H(\varepsilon_m + P) = \kappa n \psi v, \quad \dot{\varepsilon}_\psi + 6H\varepsilon_\psi = -\kappa n \psi v. \quad (6.73)$$

The control uses $\kappa = 40000$, $N_0 = 4$, $a(0) = 1$, $v(0) = 0$, the positive initial H fixed by the constraint, and $\psi(0) = 0, 0.0005, 0.002, 0.005$ over $0 \leq t \leq 0.04$. Independent evolution of H monitors rather than projects the constraint. Fixed-step refinement and an adaptive reference show fourth-order endpoint convergence. For the nonzero-scalar runs, the endpoint refinement

shows the stated fourth-order behavior; the zero-scalar case is already at the roundoff/reference-error scale. The largest amplitude changes final a from the zero-probe value 1.23542171 to 1.27117084; all runs remain at $n \geq 1.947$.

The reference relative gravitational-constraint residual is below 9×10^{-13} and integrated exchange-balance errors below 6×10^{-13} . These numerical solutions demonstrate finite coupled exchange in the homogeneous setting. They are separate from the bounded-body evolution with zero probe.

Result

The retained Einstein and matched matter laws give exact GR correspondence on the regular deformation chart. Weak forces, clock ratios, deflection, delay and precession share that same field/probe choice. Finite vacuum geometry, constrained local dynamics and specified supported-material and signal calculations extend the correspondence substantially beyond those weak tests.

Notes

A different finite field law or internal source, material, clock or signal coupling can change predictions and must be tested with its own reciprocal source and dynamics. Agreement of weak coefficients does not uniquely select those laws. The internally compensated native particle, universal measuring standards and a complete positive curved-system ADMC account remain to be derived. The mainline synthesis is §3.10.

Derivation 4.3A — Quantum directional readings and coherent states

Purpose

Six opposed momentum readings describe a carrier's core and translation, but they do not replace its quantum state. This derivation supports proposed §4.3, “Phase-bearing states and directional readings.” It establishes the positive free reading operators, identifies the shell information carried by their second moments, and constructs a six-reading representation that retains the complete fixed-carrier density operator rather than discarding relative phase.

Starting assumptions

Work in one inertial local frame, with $x^0 = ct$ and $p_f > 0$ the fermic scale of the declared free-space realization. Retain a complex quantum Hilbert space, normalized density operator $\varrho \geq 0$, unitary free evolution and the usual Born rule for a specified measurement. None is inferred from directional conservation alone. In momentum representation,

$$\mathcal{H} = L^2(d^3p) \otimes \mathcal{H}_{\text{int}}, \quad \widehat{M}_+ = \sqrt{p_f^2 + \hat{\mathbf{p}}^2}.$$

The internal space is carried unchanged throughout. Use states with finite second moments for the moment identities; use smooth wave packets or a finite periodic regulator when writing kernels. The free massive inverse is bounded; a massless extension needs its zero-core mode and inverse domain treated separately.

Positive readings on common support

Let $A = (i, \sigma)$ abbreviate the native slashed directional index $\ni$, with $i = 1, 2, 3$ and $\sigma = \pm 1$. Define

$$\hat{p}^{i\sigma} = \widehat{M}_+ + \frac{\sigma}{2} \hat{p}_i.$$

All four underlying operators are multiplication operators in momentum space. Their common spectral domain makes the sums and products unambiguous. Since $M(p) = \sqrt{p_f^2 + |\mathbf{p}|^2} \geq |\mathbf{n} \cdot \mathbf{p}|$ for every unit $\mathbf{n}$,

$$\widehat{M}_+ \pm \frac{1}{2} \mathbf{n} \cdot \widehat{\mathbf{p}} \geq \frac{1}{2} \widehat{M}_+ \geq 0.$$

These inequalities are quadratic-form inequalities, obtained pointwise in the joint spectral representation. In particular,

$$\widehat{p}^{i+} + \widehat{p}^{i-} = 2\widehat{M}_+, \quad \widehat{p}_i = \widehat{p}^{i+} - \widehat{p}^{i-}, \quad \widehat{M}_+ = \frac{1}{6} \sum_A \widehat{p}^A.$$

The factor 1/2 multiplies the signed translation, not the entire opposed reading. The six entries sum to six times the same core; they are not six populations or six independently additive budgets.

Retain Gravity's indexed representation maps,

$$L^{i\sigma}_0 = 1, \quad L^{i\sigma}_j = \frac{\sigma}{2} \delta_{ij}, \quad L^0_{i\sigma} = \frac{1}{6}, \quad L^j_{i\sigma} = \sigma \delta^j_i.$$

Multiplication gives $L^\mu_A L^A_\nu = \delta^\mu_\nu$, whereas the reverse product is

$$\Pi^{i\sigma}_{j\tau} = \frac{1}{6} + \frac{\sigma\tau}{2} \delta_{ij}, \quad \Pi^2 = \Pi.$$

Its image consists exactly of lists with equal opposed pair sums. It has rank four: a common component and three opposed differences. For a fixed p_f , the future shell within that support has three independent momentum parameters. Neither count specifies spin states.

With local signature $\eta_{\mu\nu} = \text{diag}(-1, 1, 1, 1)$, the supported forms are

$$\eta_{i\sigma, j\tau} = \sigma\tau \delta_{ij} - \frac{1}{36}, \quad \eta^{i\sigma, j\tau} = -1 + \frac{\sigma\tau}{4} \delta_{ij}, \quad \eta^{AC} \eta_{CB} = \Pi^A_B.$$

They compare the common core with the opposed differences; they are not a positive quantum norm. Direct substitution proves

$$\eta_{AB} \widehat{p}^A \widehat{p}^B = -\widehat{M}_+^2 + \widehat{\mathbf{p}}^2 = -p_f^2 I.$$

Positivity does not replace this shell condition. For example, $(M, p_1, p_2, p_3) = (1, 3/2, 0, 0)$ produces six positive supported entries but violates $M \geq |\mathbf{p}|$. Conversely, adding a redundant

pair-even vector can leave the invariant unchanged while violating support. Both support and the shell must therefore be imposed.

The shell belongs to second moments, not generally to means

Define the reading means and symmetric covariance by

$$\mu^A = \text{Tr}(\varrho \hat{p}^A), \quad \Sigma^{AB} = \frac{1}{2} \langle \{ \hat{p}^A - \mu^A, \hat{p}^B - \mu^B \} \rangle.$$

Taking the expectation of the operator identity, then separating the product into mean and covariance, gives

$$\boxed{\eta_{AB}(\mu^A \mu^B + \Sigma^{AB}) = -p_f^2.}$$

Equivalently,

$$\langle M \rangle^2 - |\langle \mathbf{p} \rangle|^2 = p_f^2 + \sum_i \text{Var}(p_i) - \text{Var}(M).$$

The rightmost terms are momentum spread, not a changed fermic identity. The square-root function satisfies $|M(p) - M(q)| \leq |p - q|$; averaging its square over two independent samples proves $\text{Var}(M) \leq \sum_i \text{Var}(p_i)$. Thus the mean four-vector remains future causal, but generally lies above the fixed one-carrier shell.

For a concrete example take $p_f = 1$ and equal weights at $p_x = \pm\sqrt{3}$, with the other components zero. Both modes have $M = 2$. All six means are 2, so the shell inferred from means alone would give $\langle M \rangle^2 = 4$. The actual shell has $p_f^2 = 1$; the missing 3 is $\text{Var}(p_x)$. A coherent superposition and the incoherent mixture have the same reading moments here. This identity is neither a coherence witness nor evidence that spread changes p_f .

In a static zero-shift reference comparison, all readings instead scale as $p_r^A = N p^A$. At a fixed common N , their shell is $\eta_{AB} p_r^A p_r^B = -p_{f,r}^2$, with $p_{f,r} = N p_f$. This is a different comparison of the same shell, not a new local coefficient.

An explicit phase difference invisible to every reading statistic

In a periodic interval of length $\mathcal{L}$, choose a nonzero allowed wave number $k = 2\pi n/\mathcal{L}$ and the scalar Fourier representative

$$\psi_{\vartheta}(x) = \frac{1 + e^{i\vartheta} e^{ikx}}{\sqrt{2\mathcal{L}}}.$$

The two orthogonal momentum modes 0 and $\hbar k$ each have probability 1/2. Therefore every function of the commuting momentum readings has the same spectral distribution for all ϑ . Yet a position measurement in this specified representation gives

$$|\psi_{\vartheta}(x)|^2 = \frac{1 + \cos(kx + \vartheta)}{\mathcal{L}}.$$

Changing ϑ translates the interference pattern. Replacing the state by reading means, by all diagonal moments, or even by the entire joint reading distribution loses that change. This example uses a stated Fourier position readout; it does not claim a unique relativistic localization prescription.

Bilocal reconstruction and its normalization

The missing information is retained if both momentum arguments and the internal indices survive. Write $y^A(p) = M(p) + \sigma p_i/2$ only as a calculation alias for the eigenvalue of $\hat{p}^A$, and set

$$f_A(p) = \frac{y^A(p)}{\sqrt{M(p)}}, \quad \mathcal{C}_{ss'}^{AB}(p, p') = f_A(p) \varrho_{ss'}(p, p') f_B(p').$$

Since $\sum_A f_A(p) = 6\sqrt{M(p)}$, summing both reading indices gives the inverse immediately:

$$\boxed{\varrho_{ss'}(p, p') = \frac{\sum_{A,B} \mathcal{C}_{ss'}^{AB}(p, p')}{36\sqrt{M(p)M(p')}}.}$$

Thus the full kernel is injective, including off-diagonal phases and all internal coherence. As an operator, $\mathcal{C} = F\varrho F^\dagger$ is positive on its form domain. This does not make its individual off-diagonal entries real or positive.

Its trace, however, is not one in general. The extra weight is

$$d(p) = \sum_A f_A(p)^2 = 6M(p) + \frac{|\mathbf{p}|^2}{2M(p)}.$$

To preserve the quantum norm directly, define

$$u_A(p) = \frac{y^A(p)}{\sqrt{6M(p)^2 + |\mathbf{p}|^2/2}}, \quad (U\psi)_A(p) = u_A(p)\psi(p).$$

Here the displayed u_A is a real dimensionless amplitude coefficient, not a velocity. The sum $\sum_A u_A^2 = 1$ proves $U^\dagger U = I$, so

$$\varrho_6 = U\varrho U^\dagger, \quad \text{Tr } \varrho_6 = 1, \quad \mathcal{C} = \sqrt{d} \varrho_6 \sqrt{d}.$$

At each fixed momentum its range projector is $P_{AB}(p) = u_A(p)u_B(p)$, of rank **one in reading space**, tensored with the unchanged internal space. The condition $\Pi\Xi = \Xi$ alone is too weak for an amplitude Ξ : it would allow extra components absent from the original state. The actual condition is $P\Xi = \Xi$. Rank-four classical support and rank-one fixed-momentum amplitude support answer different questions.

For a fixed encoding and a specified self-adjoint generator, the evolution also lifts without adding physics:

$$\widehat{M}_6 = U\widehat{M}_t U^\dagger \quad \text{on } \text{Ran } P, \quad i\hbar\partial_0\varrho_6 = [\widehat{M}_6, \varrho_6].$$

The inverse $U^\dagger$ recovers the original evolution. Extending this operator to the forbidden complement and calling its additional zero modes physical would change the state space, not merely its notation.

Result and return to the chapter

The free directional operators are positive, supported observables. Their second moments obey the fixed-carrier shell; their diagonal statistics do not determine phase. A correctly normalized bilocal six-reading encoding retains the full original state, without adding six spin or particle modes. Proposed §4.3 can therefore use readings and moments as useful observables while keeping the coherent state indispensable. Neither the encoding nor its kinetic diagonal selects an exact local gravitational source, and a fixed-carrier density operator is not a complete field state with variable particle number.

Derivation 4.4A — Core momentum and first-order evolution

Purpose

A positive core magnitude and a local first-order spinor generator are related, but they are not the same operator on every state. This derivation supports proposed §4.4, “The core momentum operator.” It exposes the factorization premise, identifies the free positive spectral sector, and rewrites the chosen spinor equation in constrained directional indices without creating six spin states.

Starting assumptions

Retain the complex state space, Born readout and continuous unitary time evolution of ordinary quantum mechanics. In a flat inertial frame let $\hat{p}_i = -i\hbar\partial_i$, with a common smooth core and the self-adjoint free realization on $H^1(\mathbb{R}^3, \mathbb{C}^4)$ or a periodic spatial domain. The fermic scale $p_f > 0$ belongs to the declared free-space identity realization. Introduce constant Hermitian matrices obeying

$$\beta^2 = I, \quad \{\beta, \alpha^i\} = 0, \quad \{\alpha^i, \alpha^j\} = 2\delta^{ij}I.$$

A concrete choice is

$$\beta = \begin{pmatrix} I_2 & 0 \\ 0 & -I_2 \end{pmatrix}, \quad \alpha^i = \begin{pmatrix} 0 & \sigma_i \\ \sigma_i & 0 \end{pmatrix},$$

where σ_i are the Pauli matrices. This is a stipulated spinor realization of the shell, not a consequence of ADMC alone. The historical first-order construction is Dirac’s (Dirac 1928).

The Clifford square and the two spectral signs

Define the signed flat generator

$$D = \beta p_f + \alpha^i \hat{p}_i.$$

Its square can be checked without choosing a matrix basis:

$$\begin{aligned} D^2 &= \beta^2 p_f^2 + p_f \{\beta, \alpha^i\} \hat{p}_i + \frac{1}{2} \{\alpha^i, \alpha^j\} \hat{p}_i \hat{p}_j \\ &= (p_f^2 + \hat{\mathbf{p}}^2) I. \end{aligned}$$

The antisymmetric matrix part drops out only because the free momentum components commute. The spectral magnitude is therefore

$$|D| = \widehat{M}_+ I, \quad \widehat{M}_+ = \sqrt{p_f^2 + \hat{\mathbf{p}}^2}.$$

At each momentum D has eigenvalues $\pm M(p)$, each of multiplicity two in this realization. Because $p_f > 0$, the projectors

$$P_{\pm} = \frac{1}{2} \left(I \pm \frac{D}{\widehat{M}_+} \right)$$

are well defined. The square identity proves $P_{\pm}^2 = P_{\pm}$, $P_+ P_- = 0$, and $D P_{\pm} = \pm \widehat{M}_+ P_{\pm}$. Thus D agrees with the positive core operator **on** $\text{Ran } P_+$; it is not positive on the entire four-spinor space.

The square-root formulation keeps the positive free spectrum explicitly. In position space its Fourier multiplier is not a polynomial in momentum, so it is generally a nonlocal pseudodifferential operator. The full D is local and first order, at the cost of the larger signed spinor representation. Projection back onto P_+ is itself momentum dependent and generally spatially nonlocal. One cannot retain an arbitrary locally supported four-spinor while simultaneously claiming it belongs entirely to the positive spectral sector.

This distinction does not assign negative physical core to antimatter. The two fermic-cycle orientations retain positive physical core in the native interpretation; their relation to field coefficients requires a field representation and is not supplied by relabeling a negative eigenvalue.

What the phase law assumes and what it yields

For an autonomous self-adjoint $\widehat{M}_t$, retain the phase-to-generator calibration

$$U(t) = \exp(-ic\widehat{M}_t t/\hbar).$$

Differentiating on its generator domain gives

$$\boxed{\frac{i\hbar}{c}\partial_t\psi = \widehat{M}_t\psi.}$$

A mode with generator value m_t has phase $e^{-icm_t t/\hbar}$ and angular frequency $cm_t/\hbar$. In the flat full-spinor realization $\widehat{M}_t = D$; within its invariant positive sector the same equation uses $\widehat{M}_+$. Multiplying by c gives the conventional energy-unit generator $\widehat{H} = c\widehat{M}_t$, with $cp_f = m_0c^2$.

Continuous unitary evolution supplies a self-adjoint generator. The identification of that generator with this shell realization, its phase calibration by $\hbar$, the quantum state grammar and the measurement rule remain retained premises. The algebra does not derive them from conservation of classical directional totals. Nor does a first-order time derivative alone imply a particular microscopic spin or statistics law.

One Clifford convention in four and six displays

Keep Gravity's signature $\eta = \text{diag}(-1, 1, 1, 1)$, and choose

$$\gamma^0 = \beta, \quad \gamma^i = \beta\alpha^i, \quad \{\gamma^\mu, \gamma^\nu\} = -2\eta^{\mu\nu}I.$$

Thus $(\gamma^0)^2 = I$ and $(\gamma^i)^2 = -I$. The minus sign in the Clifford relation does not alter the interval signature. Multiplication of $(i\hbar\gamma^\mu\partial_\mu - p_f)\Psi = 0$ by β gives $i\hbar\partial_0\Psi = D\Psi$ directly.

Use $A = (i, \sigma)$ as the short slashed-index label and the indexed L maps of Derivation 4.3A. The upper directional matrices and supported derivatives are

$$\gamma^A = L^A{}_\mu \gamma^\mu, \quad \partial_A = L^\mu{}_A \partial_\mu.$$

Since the return followed by the lift is the four-component identity,

$$\gamma^A \partial_A = \gamma^\mu \partial_\mu, \quad \{\gamma^A, \gamma^B\} = -2\eta^{AB}I.$$

For a future on-shell momentum $p^\mu = (M, p_i)$ it is useful instead to form the lower coefficients

$$K_A = -\eta_{AB}\gamma^B, \quad K_{i\sigma} = \frac{1}{6}\gamma^0 - \sigma\gamma^i.$$

The opposed sums and differences then give

$$\begin{aligned} K_A p^A &= \gamma^0 M - \gamma^i p_i, & \{K_A, K_B\} &= -2\eta_{AB}I, \\ (K_A p^A)^2 &= -\eta_{AB} p^A p^B I = p_f^2 I. \end{aligned}$$

Consequently $(K_A p^A - p_f)u = 0$ is precisely the original positive-plane-wave spinor equation. For the differential equation, define $\hat{p}^A = -i\hbar\eta^{AB}\partial_B$; support gives $K_A \hat{p}^A = i\hbar\gamma^\mu\partial_\mu$. This four-derivative notation is not the commuting positive spectral reading operator on arbitrary spacetime-dependent spinors. The two agree on the specified positive plane waves, where $\partial_0 = -iM/\hbar$ and $\partial_i = ip_i/\hbar$.

Each matrix still acts on the original four-spinor. Their six labels satisfy linear dependencies; they are not six independent Clifford generators. In a varying frame the coframe and compatible spin lift must be included, rather than promoting the six labels to unrelated partial derivatives.

Result and return to the chapter

The chosen first-order algebra factors the free shell and supplies a local signed generator. Its positive spectral restriction recovers the square-root core; its directional display preserves the original spinor and derivatives. Proposed §4.4 should retain the short squaring argument and its premise, while using this derivation for the spectral and representation distinctions. Spin, statistics, the Born rule and native microscopic coupling are not selected by this factorization.

Derivation 4.5A — Coframe evolution and probability conservation

Purpose and assumptions

A varying Stage changes spatial derivatives, spin transport and the measure used to normalize a state. This derivation supports proposed §4.5, “Quantum motion in deformed space,” and the changing-Stage boundary in proposed §4.9. It derives a complete prescribed-background spinor control, including the terms that a shell calculation alone cannot fix.

Assume an oriented, torsion-free, metric-compatible spin geometry with a smooth prescribed Stage-adapted coframe, $\phi^0_i = 0$, $N = \phi^0_0 > 0$. Retain the minimally coupled spinor equation and the Clifford convention of Derivation 4.4A. The background is not varied dynamically here. Define calculation aliases

$$E^a_i = \phi^a_i, \quad w^i = (E^{-1})^i_a \phi^a_0, \quad h_{ij} = \delta_{ab} E^a_i E^b_j, \quad W = \det E > 0.$$

Then

$$\begin{aligned} d\ell^0 &= N dx^0, & d\ell^a &= E^a_i (dx^i + w^i dx^0), \\ ds^2 &= -N^2 (dx^0)^2 + h_{ij} (dx^i + w^i dx^0)(dx^j + w^j dx^0). \end{aligned}$$

Here h^{ij} means the inverse **spatial** metric, not the spatial block of the inverse spacetime metric. Physical spatial volume is $W d^3x$. Coordinate momentum is $P_i = p_{\text{coord},i}$, and $p_{\text{local},a} = (E^{-1})^i_a P_i$.

Construct the connection before solving for evolution

Let $\Gamma^\rho_{\mu\nu}$ be the Levi-Civita connection of $g = \phi^\top \eta \phi$. In matrix notation $(\Gamma_\mu)^\rho_\nu = \Gamma^\rho_{\mu\nu}$. Coframe compatibility fixes the local frame connection:

$$\mathcal{A}_\mu = (\phi \Gamma_\mu - \partial_\mu \phi) \phi^{-1}.$$

Its Lorentz-lowered pair is antisymmetric. For the convention $\{\gamma^a, \gamma^b\} = -2\eta^{ab}I$, the compatible spin lift is

$$A_\mu = -\frac{1}{4}\eta_{ac}A_\mu{}^c{}_b\gamma^a\gamma^b, \quad D_\mu = \partial_\mu + A_\mu.$$

Indeed the Clifford commutator identity yields

$$\partial_\mu\gamma^\nu + \Gamma^\nu{}_{\mu\lambda}\gamma^\lambda + [A_\mu, \gamma^\nu] = 0, \quad \gamma^\mu = (\phi^{-1})^\mu{}_a\gamma^a.$$

This supplies an explicit construction and a check of every sign. It is not an independent gravitational field fitted after the generator is written.

Since $\gamma^0 = \beta/N$, define the Hermitian velocity matrices

$$A^i = N\alpha^a(E^{-1})^i{}_a - w^iI = N\beta\gamma^i.$$

Multiplying the stipulated equation $(i\hbar\gamma^\mu D_\mu - p_f)\Psi = 0$ by $N\beta$ gives

$$i\hbar\partial_0\Psi = \widehat{M}_{t,\text{raw}}\Psi, \\ \widehat{M}_{t,\text{raw}} = A^i\hat{P}_i + \beta N p_f - i\hbar(A_0 + A^i A_i), \quad \hat{P}_i = -i\hbar\partial_i.$$

The shift is an identity-valued drift $-w^i P_i$, together with its connection effects. It is not generally a fixed momentum subtraction inside the α term.

Put the changing norm into a fixed Hilbert space

The slice norm is $\int W\Psi^\dagger\Psi d^3x$. Set $\psi = W^{1/2}\Psi$. Differentiating both $W^{-1/2}\psi$ in space and $W^{1/2}\Psi$ in time gives

$$\boxed{\widehat{M}_t = A^i\hat{P}_i + \beta N p_f + Z,}$$

$$\boxed{Z = -i\hbar(A_0 + A^i A_i) + \frac{i\hbar}{2}(A^i\partial_i \ln W + I\partial_0 \ln W).}$$

Evolution is $i\hbar\partial_0\psi = \widehat{M}_t\psi$, now with norm $\int \psi^\dagger\psi d^3x$. The temporal derivative is essential whenever the spatial measure changes. This is the changing-inner-product issue treated in the curved-spinor literature, for example by Huang and Parker.

Equivalently,

$$\widehat{M}_t = \frac{1}{2}\{A^i, \widehat{P}_i\} + \beta N p_f + V_{\text{spin}}, \quad V_{\text{spin}} = Z + \frac{i\hbar}{2}\partial_i A^i.$$

The spin potential is fixed by the compatible lift. Symmetrizing the derivative term by hand without this remainder is not the general coframe equation.

Continuity, adjointness and the boundary term

The spinor equation and its adjoint give $\nabla_\mu j_{\text{prob}}^\mu = 0$, where $j_{\text{prob}}^\mu = \bar{\Psi}\gamma^\mu\Psi$ and $\bar{\Psi} = \Psi^\dagger\beta$. Since $\sqrt{-g} = NW$,

$$\sqrt{-g} j_{\text{prob}}^0 = W\Psi^\dagger\Psi, \quad \sqrt{-g} j_{\text{prob}}^i = W\Psi^\dagger A^i\Psi.$$

Therefore

$$\partial_0(\psi^\dagger\psi) + \partial_i(\psi^\dagger A^i\psi) = 0.$$

In ordinary time the probability flux is $c\psi^\dagger A^i\psi$. It is not the gravitational source tensor.

The same identity follows by subtracting the generator equation and its conjugate. The required coefficient relation is

$$Z - Z^\dagger = -i\hbar\partial_i A^i.$$

For test spinors u, v in a spatial region Ω , integration by parts now gives the exact Green identity

$$\langle u, \widehat{M}_t v \rangle - \langle \widehat{M}_t u, v \rangle = -i\hbar \int_{\partial\Omega} u^\dagger A^i v n_i dS.$$

The volume terms cancel; formal adjointness therefore requires a boundary domain, not merely real eigenvalues of the shell symbol. In the raw representation the corresponding coordinate-measure identity is instead

$$\widehat{M}_{t,\text{raw}}^\dagger W - W\widehat{M}_{t,\text{raw}} = i\hbar\partial_0 W.$$

To see the sign, differentiate $\langle \Psi, W\Psi \rangle$: the two evolution terms contribute $(i/\hbar)\langle \Psi, (M_{t,\text{raw}}^\dagger W - W M_{t,\text{raw}})\Psi \rangle$, which cancels $\langle \Psi, \partial_0 W\Psi \rangle$. Setting the raw adjoint defect to zero while W changes would violate norm conservation.

Three explicit controls

Static isotropic space

Take $E = b(\mathbf{x})I$, $w = 0$ and $F = N/b$. The scalar b is a spatial scale, distinct from the indexed four/six maps L . For variation along x^1 , the nonzero spin coefficients are

$$A_0 = \frac{N'}{2b}\alpha^1, \quad A_2 = \frac{b'}{2b}\gamma^1\gamma^2, \quad A_3 = \frac{b'}{2b}\gamma^1\gamma^3.$$

Thus $A_0 + A^i A_i = (N'/(2b) + Nb'/b^2)\alpha^1$. Combining this with $\partial_1 \ln W = 3b'/b$ yields $Z = -i\hbar\alpha^1 F'/2$. The vector form is

$$\widehat{M}_t = \beta N p_f + \frac{1}{2}\{F, \alpha \cdot \widehat{\mathbf{P}}\}.$$

Its full semiclassical dispersion matrix, including the zeroth-order fermic term, squares to $N^2(p_f^2 + |\mathbf{P}|^2/b^2)I$. This is not the derivative-only principal symbol used to define characteristic cones. Its positive eigenvalue is $M_t = NM = M_r$, with $\mathbf{p}_r = F\mathbf{P}$ and $p_{f,r} = Np_f$. Both slots use the same comparison. This control agrees with the Hermitian isotropic construction of Obukhov, Silenko and Teryaev.

A rotating flat-coordinate frame

Take $N = 1$, $E = I$, $w = \Omega \times \mathbf{x}/c$, with constant Ω . For rotation around z , write $\omega = \Omega/c$. The compatible spin lift gives $A_0 = -i\omega\Sigma_z/2$ and $A_i = 0$, where $\Sigma_z = \text{diag}(\sigma_z, \sigma_z)$. Hence

$$\widehat{M}_t = \beta p_f + \alpha \cdot \widehat{\mathbf{P}} - \frac{1}{c}\Omega \cdot (\mathbf{L}_{\text{orb}} + \mathbf{S}), \quad \mathbf{S} = \frac{\hbar}{2}\Sigma.$$

The shift supplies the orbital term and the connection supplies spin rotation. The geometry is flat: this checks frame transport, not a sourced rotating gravitational field. A timelike rotating laboratory lies inside its light cylinder; no global positivity claim for the rotating generator is required.

A homogeneous changing Stage

For $N = 1$, $w = 0$, $E = \text{diag}(a_1, a_2, a_3)$, let $H_i = \partial_0 \ln a_i$. The compatible lift has $\mathbf{A}_0 = 0$, $\mathbf{A}_i = (\partial_0 a_i) \alpha^i / 2$. Consequently $A^i \mathbf{A}_i = \sum_i H_i I / 2$, and its contribution to Z cancels $i\hbar \partial_0 \ln W / 2$ exactly. The result is

$$\widehat{M}_t = \beta p_f + \sum_i \alpha^i \widehat{P}_i / a_i.$$

For $a_i = a$, identify Expansion's supplied yield $\chi = 1/a > 0$:

$$\widehat{M}_\chi = \beta p_f + \chi \alpha \cdot \widehat{\mathbf{P}}, \quad M_\chi = \sqrt{p_f^2 + \chi^2 |\mathbf{P}|^2}.$$

Spatial homogeneity conserves the canonical labels P_i . A narrow packet following an instantaneous branch has $\dot{x}^i = c \chi^2 P_i / M_\chi$. The factor χ^2 follows by differentiating the branch magnitude, not from an additional law. The background history remains supplied, not solved here.

Bounded unitary evolution and the positive-sector boundary

A clean existence statement uses a finite time interval and a compact spatial slice without boundary, for example a periodic control. Assume smooth coefficients with bounded derivatives, N bounded away from zero, and uniformly nondegenerate E . The rescaled equation is a linear symmetric hyperbolic system with time matrix I . Sobolev Cauchy existence, uniqueness and continuous dependence supply forward and backward evolution for smooth data. The proved continuity law preserves its L^2 norm. Extension by density and backward evolution give an onto isometry, hence a unitary propagator of norm one. "Bounded" here describes the propagator and time interval, not the generally unbounded differential generator.

For stationary coefficients this is a strongly continuous unitary group. If also $|w|_h < N$, the derivative symbol has no zero eigenvalue at nonzero covector, so it is elliptic; on the smooth compact slice its self-adjoint realization has domain H^1 . With a physical boundary one must specify a maximal admissible boundary domain. Requiring zero flux of each individual state alone is not such a specification. MIT-type reflecting boundary problems are an established example under their own hypotheses; see Große and Murro. Noncompact falloff, singular charts and nonlinear backreaction are outside this bounded statement.

Unitarity does not imply preservation of an instantaneous positive spectral subspace. For one homogeneous momentum mode and a polarization block, write

$$h_j = p_f \sigma_z + \chi_j P \sigma_x, \quad \epsilon_j = \sqrt{p_f^2 + \chi_j^2 P^2}.$$

At a sudden change the spinor is continuous. Projecting the old positive state onto the new negative subspace gives

$$\begin{aligned} P_{+\rightarrow-} &= \text{tr} \left[\frac{I - h_2/\epsilon_2}{2} \frac{I + h_1/\epsilon_1}{2} \right] \\ &= \frac{1}{2} \left[1 - \frac{p_f^2 + \chi_1 \chi_2 P^2}{\epsilon_1 \epsilon_2} \right]. \end{aligned}$$

The quench is a separate piecewise-constant control, not a smoothness premise of the Cauchy theorem. For smooth change set $\tan \vartheta = \chi P/p_f$. Rotation to the instantaneous basis supplies an off-diagonal term of magnitude $\hbar|\partial_0 \vartheta|/2$, against gap 2ϵ . Thus the local adiabatic diagnostic is

$$\partial_0 \vartheta = \frac{p_f P \partial_0 \chi}{p_f^2 + \chi^2 P^2}, \quad \frac{\hbar|\partial_0 \vartheta|}{4\epsilon} \ll 1.$$

A controlled branch approximation also needs a nonclosing gap, sufficiently regular slow history and control of accumulated transitions over the interval; a small instantaneous ratio alone is not a global error theorem. For $p_f = 0$, fixed nonzero P and positive χ , the eigendirection is constant and this mixing vanishes. Neither example identifies a massless spinor with a photon or derives particle creation without a field-state prescription.

Result and return to the chapter

The prescribed minimal spinor control has an explicit generator, conserved probability flux and a bounded unitary Cauchy evolution under stated domain hypotheses. It matches the static local/reference shell and Expansion's homogeneous operator. Proposed §§4.5 and 4.9 can use those results while distinguishing full-spinor evolution from branch-following approximations. The construction does not select native microscopic coupling or a self-consistent quantum Stage law.

Derivation 4.6A — Path phase and coherent clocks

Purpose

Phase accumulated along a carrier's path and a clock's observable recurrence are different quantities. This derivation supports proposed §4.6, "Phase and physical clocks." It first relates a WKB path phase to the ideal-cycle expression inherited from Gravity. It then specifies a coherent two-state clock and proves the condition under which every accessible internal process shares one rescaling of time.

Starting assumptions and the ray phase

Use the free massive local shell and prescribed minimally coupled coframe control of Derivation 4.5A. For a WKB state $\Psi = ae^{iS/\hbar}$, assume background and amplitude variations are small on the local wavelength scale, away from turning points, caustics and branch crossings. Let $P_i = \partial_i S$ and choose the future branch

$$M = \sqrt{p_f^2 + h^{ij}P_iP_j}, \quad M_t = NM - w^iP_i, \quad \partial_t S = -cM_t.$$

The ray velocity is

$$\dot{x}^i = c \frac{\partial M_t}{\partial P_i} = c \left(\frac{Nh^{ij}P_j}{M} - w^i \right).$$

Evaluate the **total** phase derivative along that ray rather than at a fixed coordinate:

$$\begin{aligned} \frac{dS}{dt} &= P_i \dot{x}^i - cM_t \\ &= cN \frac{h^{ij}P_iP_j}{M} - cw^iP_i - cNM + cw^iP_i \\ &= -\frac{cNp_f^2}{M}. \end{aligned}$$

The shift cancels between transport and the generator. Substituting the same ray velocity into the interval gives

$$\frac{d\tau}{dt} = \sqrt{N^2 - h_{ij}(\dot{x}^i/c + w^i)(\dot{x}^j/c + w^j)} = \frac{Np_f}{M}.$$

Hence

$$\boxed{\frac{dS}{dt} = -cp_f \frac{d\tau}{dt}}.$$

This is local along the ray and does not require a conserved M_t . In a static zero-shift comparison, $M_r = NM$ and $p_{f,r} = Np_f$, so

$$-\frac{1}{\hbar} \frac{dS}{dt} = \frac{c}{\hbar} \frac{p_{f,r}^2}{M_r}.$$

It matches the magnitude of Gravity's stipulated ideal-cycle rate. At a fixed coordinate, by contrast, the phase frequency is $cM_t/\hbar$. Confusing the partial and total derivatives would lose the translational subtraction. For a null ray the leading action is constant along the ray although fixed-location waves can oscillate; no fermic internal clock is inferred for it.

A common phase multiplying an isolated state cancels from all its expectation values. The identity above therefore does not, by itself, give a detector-readable clock. An observable recurrence needs at least a relative phase and a specified measurement.

A complete two-state clock control

Consider a small held internal system whose support does not change its internal dynamics and across which N is effectively uniform. Assume its local momentum-unit generator couples to proper time. With normalized internal state $|\psi\rangle$, the action is

$$\mathcal{A}_{\text{int}} = \int dt \left[\frac{i\hbar}{2} \langle \psi | \overleftrightarrow{\partial}_t | \psi \rangle - cN(t) \langle \psi | \widehat{M}_{\text{int}} | \psi \rangle \right].$$

Variation, with normalization enforced if required, gives $i\hbar\partial_t|\psi\rangle = cN\widehat{M}_{\text{int}}|\psi\rangle$ up to an irrelevant common-phase term. This is an explicit proper-time coupling premise; the WKB identity has not derived the internal action.

Choose

$$\widehat{M}_{\text{int}} = M_0 I + g \sigma_x, \quad M_0 > g > 0, \quad |\psi(0)\rangle = |L\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$

The eigenvalues $M_0 \pm g$ are positive. Define $\tau(t) = \int_0^t N(t') dt'$ and $\vartheta = cg\tau/\hbar$. Since the fixed internal generator commutes with itself at every time,

$$|\psi(t)\rangle = e^{-icM_0\tau/\hbar} (\cos \vartheta |L\rangle - i \sin \vartheta |R\rangle).$$

A population measurement gives

$$\boxed{P_R(t) = \sin^2 \vartheta, \quad \langle \sigma_z \rangle = \cos(2\vartheta).}$$

The common M_0 phase disappears; the gap $2g$ generates the measurable oscillation. At constant lapse the reference period is $\pi\hbar/(cNg)$ and the proper period is $\pi\hbar/(cg)$. Uniform time dependence of N does not require an adiabatic approximation in this model. Repeated readout would need a non-disruptive protocol or identically prepared trials; the formula is the coherent free-running signal between measurements.

Universal rescaling on an accessible sector

Now ask a stronger question than whether one clock slows: when do **all** states and observables in an accessible internal sector undergo the same time change? In a finite-dimensional irreducible sector, require the infinitesimal dynamics to satisfy

$$[\widehat{M}_r, \varrho] = \Lambda[\widehat{M}_0, \varrho] \quad \text{for every density operator } \varrho.$$

Set $D = \widehat{M}_r - \Lambda\widehat{M}_0$. Density operators span the Hermitian matrices, and their complex span is the full matrix algebra. Therefore D commutes with every matrix unit. Commuting with $|j\rangle\langle j|$ removes the off-diagonal entries of D . Commuting with $|j\rangle\langle k|$ makes its diagonal entries equal. It follows that

$$\boxed{\widehat{M}_r = \Lambda\widehat{M}_0 + aI.}$$

Conversely the added scalar commutes with every state, so this identity immediately proves the desired dynamics. For fixed $\widehat{M}_0$ and time-dependent $\Lambda(t)$, the same argument yields evolution under the accumulated parameter $\int \Lambda(t) dt$, with only a common phase from $a(t)$.

For genuine superselection sectors, the accessible algebra is block diagonal and the undetectable difference can be a different scalar in each irreducible block. More generally the difference belongs to the commutant of the accessible algebra. If coherences between two blocks can actually be prepared or measured, their relative scalar is no longer invisible. Infinite-dimensional versions additionally require shared domains; no unrestricted unbounded-operator theorem is assumed here.

This criterion tests all transition gaps **and** coherent couplings. Equal scaling of one population rate or one transition frequency is insufficient.

Open systems and complete events

For a time-homogeneous Markovian density evolution $\partial_t \varrho = \mathcal{L} \varrho$, equality of all dynamical maps after $t \mapsto \Lambda t$ is equivalent to

$$\boxed{\mathcal{L}_r = \Lambda \mathcal{L}_0}$$

on the accessible operator space. Necessity follows by differentiating the maps at zero time; sufficiency follows by exponentiation. This compares the **whole** superoperator, including coherent and dissipative parts, not a particular nonunique set of Lindblad operators. Time-dependent histories require the corresponding time-reparameterized generator at each time; non-Markovian processes require comparison of their full memory-dependent maps.

A fixed proper pulse illustrates why complete events matter. If a local coupling g acts for $\Delta\tau$, then in a static reference $g_r = Ng$ and $\Delta t = \Delta\tau/N$. Its pulse area is unchanged:

$$\vartheta = \frac{c}{\hbar} \int g_r dt = \frac{c}{\hbar} \int g d\tau.$$

The same pulse map $\mathcal{E}$ repeated at local Poisson cadence ν_0 gives the reference generator $N\nu_0(\mathcal{E} - I)$. That is a genuine uniform slowdown of this process. Holding the reference pulse duration fixed instead changes the proper event and is a different comparison.

To see explicitly why changed event probability is not enough, take $U_\vartheta = \cos \vartheta I - i \sin \vartheta \sigma_x$ and events of cadence ν . Direct expansion gives

$$\nu(U_\vartheta \varrho U_\vartheta^\dagger - \varrho) = \nu \sin^2 \vartheta (\sigma_x \varrho \sigma_x - \varrho) - i\nu \sin \vartheta \cos \vartheta [\sigma_x, \varrho].$$

The ratio of coherent to population coefficients is $\cot \vartheta$, where both are nonzero. At fixed cadence a small pulse-area reduction scales the population term quadratically and the coherent term linearly. It cannot rescale the full generator by one factor. This is a counterexample to inferring universal clock slowing from event probability alone, not a model-independent

experimental discriminator: variable pulse statistics, decoherence and actual material couplings would need their own specification.

Source response and result

The same held-system action fixes its lapse response,

$$\frac{\delta \mathcal{A}_{\text{int}}}{\delta N(t)} = -c \langle \widehat{M}_{\text{int}} \rangle.$$

It is not the derivative of a transition probability. In the clock preparation above, $\langle \sigma_x \rangle = 0$ throughout free running even though P_R depends on g . A switched drive or physical support must be included when the total source and momentum exchange are required.

Proposed §4.6 receives two distinct results: a path-phase identity for a specified free ray and an observable coherent clock for a specified internal coupling. The universal-rescaling criterion states exactly what real-material universality would require; the construction does not derive that universality for all matter.

Derivation 4.7A — Matched limits and gravitational barriers

Purpose and comparison conditions

The quantum generator must recover low-speed wave mechanics and classical rays before a tunnelling comparison can test anything beyond them. This derivation supports proposed §4.7, “Correspondence and controlled limits.” It derives those limits from the corrected spinor generator, then compares barriers with the preparation, geometry, apparatus and flux normalization held explicitly fixed.

Use the minimally coupled prescribed-background model of Derivation 4.5A, not an independently changed local fermic coefficient. An external barrier and its support are supplied classical apparatus; this is not a closed source calculation. No Klein/pair-production channel is included. The envelope, ray and exact scattering calculations below have different approximation assumptions, stated where used.

The slow free envelope

In flat space write the four-spinor in two-spinor blocks after removing the rest phase,

$$\psi = e^{-icp_f t/\hbar} \begin{pmatrix} \varphi \\ \xi \end{pmatrix}.$$

The first-order equation gives

$$\frac{i\hbar}{c} \partial_t \varphi = \sigma \cdot \widehat{\mathbf{P}} \xi, \quad \frac{i\hbar}{c} \partial_t \xi = \sigma \cdot \widehat{\mathbf{P}} \varphi - 2p_f \xi.$$

For a positive-sector envelope with $|P|/p_f \ll 1$ and time variation small compared with $cp_f/\hbar$, solve the second equation iteratively:

$$\xi = \frac{\sigma \cdot \widehat{\mathbf{P}}}{2p_f} \varphi + \text{higher-order terms}.$$

Because the free derivatives commute, $(\sigma \cdot \widehat{\mathbf{P}})^2 = \widehat{\mathbf{P}}^2$. Thus

$$i\hbar\partial_t\varphi = \frac{c\widehat{\mathbf{P}}^2}{2p_f}\varphi = -\frac{\hbar^2}{2m_0}\nabla^2\varphi, \quad p_f = m_0c.$$

The branch expansion independently checks the approximation:

$$M_+ - p_f = \frac{P^2}{2p_f} - \frac{P^4}{8p_f^3} + O(P^6/p_f^5).$$

The leading envelope has a second-order spatial derivative because elimination of the small component multiplies two first-order momentum operators, not because the relativistic generator has changed its order.

A controlled gravitational envelope and its ordering

For a static isotropic coframe $E = bI$, set $F = N/b$ and

$$q_i = \frac{1}{2}\{F, \hat{P}_i\}, \quad \mathcal{Q} = \sigma_i q_i.$$

Here q_i denotes the **reference-expressed** translation operator, not local orthonormal momentum. For a stationary mode with generator value C , the two block equations are

$$(C - Np_f)\varphi = \mathcal{Q}\xi, \quad (C + Np_f)\xi = \mathcal{Q}\varphi.$$

Where the multiplication operator $C + Np_f$ is bounded away from zero, exact elimination gives

$$(C - Np_f)\varphi = \mathcal{Q}(C + Np_f)^{-1}\mathcal{Q}\varphi.$$

This formula preserves the order of the varying coefficient. A local approximation $C \simeq Np_f$ inside the denominator gives $\mathcal{Q}(2Np_f)^{-1}\mathcal{Q}$ only in a region where the local kinetic excess and gradient corrections are controlled. It is not a globally valid envelope for a deep well with large variation of N .

For a weak-field envelope choose $N = 1 + \nu$, $b = 1 + \lambda$, with $\nu, \lambda = O(\varepsilon^2)$, $|P|/p_f = O(\varepsilon)$ and $\hbar/(p_f\ell) = O(\varepsilon)$ for the variation scale ℓ . Keep derivatives of the weak coefficients at the corresponding smooth scale. Then $\mathcal{Q} = \sigma \cdot \widehat{\mathbf{P}} + O(p_f\varepsilon^3)$, and after removing the constant free rest phase,

$$\frac{i\hbar}{c}\partial_t\varphi = \left[p_f\nu + \frac{\widehat{\mathbf{P}}^2}{2p_f} \right] \varphi + O(p_f\varepsilon^4).$$

Writing $\Phi = c^2\nu$ only for the Newtonian comparison gives the standard leading terms $m_0\Phi + \widehat{P}^2/(2m_0)$ in energy units. In a constant but not necessarily weak patch, the kinetic coefficient is $N/(2p_fb^2)$, not $1/(2Np_fb^2)$. A coefficient obtained by reducing only the fermic slot uses an inconsistent frame conversion.

Gradient terms are not absent from the exact operator. For example,

$$\begin{aligned} [q_i, q_j] &= -i\hbar[(\partial_i F)q_j - (\partial_j F)q_i], \\ \widehat{M}_t^2 &= N^2 p_f^2 + \sum_i q_i^2 + i\hbar p_f F \beta \alpha \cdot \nabla N \\ &\quad + \hbar \Sigma \cdot (\nabla F \times \mathbf{q}). \end{aligned}$$

These follow by applying the derivatives to smooth test spinors before simplifying. They belong to the ordered spinor control; discarding them is an approximation, not a scalar-shell identity.

WKB rays from the same equation

For $\Psi = ae^{iS/\hbar}$, the leading covariant equation is $(-\gamma^\mu \partial_\mu S - p_f)a = 0$. Multiplication by the conjugate factor and the declared Clifford relation give

$$g^{\mu\nu} \partial_\mu S \partial_\nu S = -p_f^2.$$

Its future branch in the Stage-adapted frame is

$$-\partial_0 S = M_t = N\sqrt{p_f^2 + h^{ij}P_i P_j} - w^i P_i.$$

The rays satisfy $\dot{x}^i = c\partial M_t/\partial P_i$ and $\dot{P}_i = -c\partial M_t/\partial x^i$. Thus the corrected generator returns the same declared free-carrier trajectory law. Connection terms transport amplitude and spin at the next WKB order. This approximation fails at turning points or when gradients compete with the phase scale; it is not used to obtain the exact square-barrier answer below.

Specify a matched constant-lapse barrier

Take a one-dimensional polarization block, proper distance ℓ , spatial coframe $b = 1$ and constant $N > 0$. Let $u(\ell)$ be the local additive momentum coefficient of an electrostatic barrier. Its reference contribution is Nu , so

$$\widehat{M}_t = \sigma_z N p_f + \frac{1}{2} \{N, \sigma_x \hat{P}_\ell\} + NuI.$$

Compare the **same** local incident value $\epsilon = C/N > p_f$, local height u_0 , proper width d , fermic scale and apparatus standards at each N . The stationary equation divided by N is

$$[-i\hbar\sigma_x\partial_\ell + p_f\sigma_z + u(\ell)]\psi = \epsilon\psi.$$

No lapse remains. Require $|\epsilon - u_0| < p_f$ inside the barrier, and define

$$k = \sqrt{\epsilon^2 - p_f^2}, \quad \kappa = \sqrt{p_f^2 - (\epsilon - u_0)^2}, \quad r_0 = \frac{k}{\epsilon + p_f}.$$

These k, κ have momentum units. Exterior spinors $v_\pm = (1, \pm r_0)^\top$ have local flux $\pm 2cr_0$. Unit local-flux modes are $v_\pm/\sqrt{2cr_0}$; unit reference-flux modes use $\sqrt{2cNr_0}$ instead. Identical exterior regions make the incoming/outgoing velocity factors cancel in transmission, but that cancellation must be checked rather than assumed for unequal endpoints.

Exact transfer and flux

Inside the constant barrier,

$$\partial_\ell\psi = B\psi, \quad B = \frac{1}{\hbar}[i(\epsilon - u_0)\sigma_x - p_f\sigma_y], \quad B^2 = \frac{\kappa^2}{\hbar^2}I.$$

Thus the interface-to-interface transfer is

$$Y = \cosh z I + \frac{\hbar B}{\kappa} \sinh z, \quad z = \kappa d/\hbar.$$

Let $V = (v_+, v_-)$ and $S = V^{-1}YV$. Matching at the two faces gives $S(1, r)^\top = (t, 0)^\top$, up to the irrelevant exterior propagation phase. Therefore $r = -S_{21}/S_{22}$ and $t = \det S/S_{22}$. Since $\text{tr } B = 0$, $\det S = 1$. Direct multiplication gives

$$S_{22} = \cosh z - i \frac{\epsilon(\epsilon - u_0) - p_f^2}{k\kappa} \sinh z.$$

Using

$$k^2 \kappa^2 + [\epsilon(\epsilon - u_0) - p_f^2]^2 = p_f^2 u_0^2$$

yields the exact normalized transmission,

$$T = |t|^2 = \left[1 + \frac{p_f^2 u_0^2}{k^2 \kappa^2} \sinh^2(\kappa d / \hbar) \right]^{-1}.$$

The identity $B^\dagger \sigma_x + \sigma_x B = 0$ proves $Y^\dagger \sigma_x Y = \sigma_x$, hence $R + T = 1$. This derivation includes matching and flux, not just an exponential estimate.

For $p_f = 1$, $\epsilon = 1.2$, $u_0 = 0.8$, $d = 2$ and $\hbar = 0.6$, the result is $T = 0.00512540161548$ at $N = 0.4, 0.6, 0.8, 1$. Constant lapse is a time normalization of flat geometry, so it cannot create an additional matched local effect. A different physical local interaction is a different model, not a consequence of changing reference units.

A smooth nonuniform gravitational control

Now fix the **distant** incident value $C = 1.2$ and use

$$b_a(x) = \begin{cases} \exp[1 - 1/(1 - (x/a)^2)], & |x| < a, \\ 0, & |x| \geq a, \end{cases}$$

$$N(x) = 1 - \delta b_4(x), \quad u(x) = 0.8 b_2(x), \quad p_f = 1, \quad \hbar = 0.6.$$

The bump notation b_a here denotes the stated profile; the spatial coframe remains one. Integrate from $x = -6$ to 6 , where $N = 1$ and $u = 0$. The stationary raw spinor and flux-normalized variable $f = \sqrt{N}\psi$ obey

$$\partial_x \psi = \left[B(x) - \frac{N'}{2N} I \right] \psi, \quad \partial_x f = B(x) f,$$

$$B(x) = \frac{1}{\hbar} [i(C/N - u) \sigma_x - p_f \sigma_y].$$

This $\sqrt{N}$ transformation normalizes scattering flux; it is not the spatial-volume transformation $W^{1/2}$, since $W = 1$ here. Independently integrate both matrix equations from identity initial data, transform to the same exterior V basis, and extract r, t as above. A fourth-order Runge–Kutta integration gives:

δ	T , 2,000 steps	T , 4,000 steps	Absolute change
0	0.000918942198793	0.000918942198686	1.08×10^{-13}
0.15	0.005381627568409	0.005381627568223	1.87×10^{-13}
0.35	0.709836341388783	0.709836341280029	1.09×10^{-10}

At 4,000 steps the maximum $|R + T - 1|$ is 5.26×10^{-13} ; the largest raw/flux-variable difference in T is below 9.3×10^{-14} . These numbers come from the reproduced calculation, with acceptance limits 10^{-8} for grid change and flux balance, and 10^{-9} for representation agreement. The finite matrices preserve flux analytically; their numerical residuals test the implementation. Very small grid differences can reach roundoff, so these data are an empirical convergence control, not a rigorous global truncation-error bound.

The local forbidden momentum is $\kappa_{\text{loc}}^2 = p_f^2 - (C/N - u)^2$. In a slowly varying forbidden interval, the WKB amplitude contains $\exp[-\int \kappa_{\text{loc}} d\ell/\hbar]$. A reference normal momentum would need division by N before use in that proper-distance exponent. As N varies, fixed distant preparation means a changing local incident value; substantial transmission changes are therefore possible. The numerical calculation follows the full two-component equation, including turning regions where the WKB form alone is insufficient.

Result and return to the chapter

The corrected generator recovers the Schrödinger envelope and the prescribed metric ray law. It also predicts ordinary sensitivity to a nonuniform gravitational background. The matched constant-lapse calculation gives no extra tunnelling effect from lapse dressing alone. Proposed §4.7 can retain the physical sensitivity of tunnelling without treating an unmatched normalization as a new force or a violation of local correspondence. A native additional effect would need a separately specified interaction, source and matched apparatus comparison.

Derivation 4.8A — Coherent sources and reciprocal accounting

Purpose and model boundaries

A coherent state can have positive probability and positive total core without being a positive classical carrier distribution at every point. This derivation supports proposed §4.8, “Quantum sources and reciprocal gravity.” It constructs the weighted kinetic kernel, supplies a local coherent scalar source and its directional balance, and then tests a different, explicitly specified spinor source. A finite interacting example separately demonstrates reciprocal global accounting.

Use the native readings $p^{i\sigma} = M + \sigma p_i/2$ in one declared local frame, and the indexed four/six maps L of Derivation 4.3A. Write $A = (i, \sigma)$ only as a slashed-index abbreviation. Source densities use physical spatial volume. The scalar, spinor and finite interacting examples below are distinct models; their favorable properties cannot be combined into a single unconstructed native interaction.

What the kinetic kernel retains

For commuting free positive reading operators, define

$$\hat{j}_{\text{kin}}^{AB} = \hat{p}^A \widehat{M}_+^{-1} \hat{p}^B.$$

The inverse is bounded for $p_f > 0$; products are understood on their common form domain. For any complex constants v_A ,

$$v_A^* \hat{j}_{\text{kin}}^{AB} v_B = \left(\sum_A v_A \hat{p}^A \right)^\dagger \widehat{M}_+^{-1} \left(\sum_B v_B \hat{p}^B \right) \geq 0.$$

Thus it is positive as a reading-space Gram matrix of operators. The free spectral entries are nonnegative too. Its row sum and shell contraction follow before any averaging:

$$\frac{1}{6} \sum_B \hat{j}_{\text{kin}}^{AB} = \hat{p}^A, \quad \boxed{-\eta_{AB} \hat{j}_{\text{kin}}^{AB} = p_f^2 \widehat{M}_+^{-1}}.$$

For a local classical or incoherent kinetic ensemble with density weights w_a per physical volume,

$$J_{\text{kin}}^{AB} = \sum_a w_a \frac{p_a^A p_a^B}{M_a}, \quad -\eta_{AB} J_{\text{kin}}^{AB} = \sum_a w_a \frac{p_{f,a}^2}{M_a}.$$

The inverse core belongs **inside** the moment. This is not a covariance and is not generally $p_f^2/\langle M \rangle$. The spinor probability density is a different quantity. For a static zero-shift reference comparison the same local kernel written in reference readings is

$$\frac{p^A p^B}{M} = \frac{p_r^A p_r^B}{N M_r}, \quad p_r^A = N p^A, \quad M_r = N M.$$

Changing the denominator to M_r alone would multiply the local source by N . Physical-volume conversion is a further, separate operation.

First moments also omit transport correlations. Two streams of weight w each at $\pm p \mathbf{e}_1$, core M , have $\mathcal{M} = 2wM$, $\mathcal{P}^i = 0$ and $\mathcal{C}^{11} = 2wp^2/M$. In six components,

$$J^{1+,1+} - J^{1+,1-} = \frac{wp^2}{M}, \quad J^{2+,2+} - J^{2+,2-} = 0.$$

Rotating the pair to the second axis preserves all six first moments but moves the transport. This is why the source tensor carries more information than the total readings.

Even the full diagonal kinetic distribution misses the off-diagonal phases demonstrated in Derivation 4.3A. That derivation's bilocal kernel retains the state, but a **local source** still requires its vertex: the rule that turns the two momentum arguments and their coherence into a source at an event.

A specified scalar action and its coherent vertex

Take a free scalar amplitude ψ with positive evolution $i\hbar\partial_0\psi = \widehat{M}_+\psi$ and unit spatial norm. Define $\zeta = \widehat{M}_+^{-1/2}\psi$. This is a nonlocal re-encoding of the same fixed-carrier amplitude, not an independent reservoir. Its local second-order equation follows by squaring the free generator:

$$(\partial_0^2 - \nabla^2 + p_f^2/\hbar^2)\zeta = 0.$$

Choose the covariant scalar action

$$\mathcal{A}_0 = -\frac{1}{2} \int dV \left[\hbar^2 g^{\mu\nu} \partial_\mu \zeta^* \partial_\nu \zeta + p_f^2 |\zeta|^2 \right], \quad dV = \sqrt{-g} d^4x.$$

It has action units in length coordinates. The positive-frequency condition is a restriction on its flat free initial data; it does not follow merely from this second-order equation. Define the momentum-unit source by the full on-shell metric variation,

$$\delta \mathcal{A}_0 = \frac{1}{2} \int dV J_0^{\mu\nu} \delta g_{\mu\nu}.$$

Using $\delta g^{\alpha\beta} = -g^{\alpha\mu} g^{\beta\nu} \delta g_{\mu\nu}$ and $\delta \sqrt{-g} = \sqrt{-g} g^{\mu\nu} \delta g_{\mu\nu} / 2$ gives

$$J_0^{\mu\nu} = \hbar^2 \operatorname{Re}(\partial^\mu \zeta^* \partial^\nu \zeta) + g^{\mu\nu} \mathcal{L}_0.$$

The same action therefore specifies propagation and source; the latter is not a guessed product of mean momenta. Diffeomorphism variation, with $\delta g_{\mu\nu} = 2\nabla_{(\mu} \epsilon_{\nu)}$ and the scalar equation imposed, gives $\nabla_\mu J_0^{\mu\nu} = 0$ after integration by parts for compactly supported ϵ . A coupled Stage must supply the corresponding response law and accounting; the scalar variation alone has not selected that law.

To expose the coherent source, Fourier expand ζ with amplitude $\psi(p)/\sqrt{M(p)}$ and convention $(2\pi\hbar)^{-3/2} e^{i\mathbf{p}\cdot\mathbf{x}/\hbar}$. Let $p^\mu = (M, \mathbf{p})$ and $p'^\mu = (M', \mathbf{p}')$, both on the same shell. Substitution in the bilinear source gives the four-slot vertex

$$\mathcal{V}_0^{\mu\nu}(p, p') = \frac{p^\mu p'^\nu + p'^\mu p^\nu - \eta^{\mu\nu}(p \cdot p' + p_f^2)}{2\sqrt{MM'}}.$$

The first two terms come from the two real-conjugate derivative products; the metric term comes from $\mathcal{L}_0$. Lifting both upper indices gives the native form

$$\boxed{\mathcal{V}_0^{AB}(p, p') = \frac{y^A y'^B + y'^A y^B - \eta^{AB}(y \cdot y' + p_f^2)}{2\sqrt{MM'}}},$$

where $y^A = L^A{}_\mu p^\mu$ and $y \cdot y' = \eta_{AB} y^A y'^B$. Thus for a scalar density kernel,

$$J_0^{AB}(x) = \int \frac{d^3p d^3p'}{(2\pi\hbar)^3} e^{i(\mathbf{p}-\mathbf{p}')\cdot\mathbf{x}/\hbar} \mathcal{V}_0^{AB}(p, p') \varrho(p, p'; x^0).$$

Its diagonal is $y^A y^B / M$. For $q^A = y^A - y'^A$, use $y^2 = y'^2 = -p_f^2$ to find $q \cdot y = -(p_f^2 + y \cdot y')$ and $q \cdot y' = p_f^2 + y \cdot y'$. These terms cancel the contraction of the metric term, proving

$$q_A \mathcal{V}_0^{AB} = 0.$$

That is momentum-transfer continuity of the full coherent vertex, including its off-diagonal stress.

Positive local readings with their gradient fluxes

In the flat realization set

$$v_0 = i\hbar \partial_0 \zeta = \widehat{M}_+ \zeta, \quad v_i = -i\hbar \partial_i \zeta, \quad v_f = p_f \zeta.$$

The action-derived source components are

$$u = J_0^{00} = \frac{1}{2} \left(|v_0|^2 + \sum_i |v_i|^2 + |v_f|^2 \right), \quad j_i = J_0^{0i} = \text{Re}(v_0^* v_i),$$

$$s_{ij} = J_0^{ij} = \text{Re}(v_i^* v_j) + \frac{\delta_{ij}}{2} \left(|v_0|^2 - \sum_k |v_k|^2 - |v_f|^2 \right).$$

Here u is the common local core density. Cauchy–Schwarz and $2ab \leq a^2 + b^2$ give

$$|\mathbf{j}| \leq |v_0| \sqrt{\sum_i |v_i|^2} \leq u.$$

Construct one common core first and only then its opposed readings:

$$\boxed{d^{i\pm} = u \pm \frac{1}{2} j_i \geq \frac{u}{2} \geq 0, \quad d^{i+} + d^{i-} = 2u.}$$

The scalar equation supplies their exact flux. For one real component z of ζ , write $\mathcal{E}[z] = \partial_0^2 z - \nabla^2 z + p_f^2 z / \hbar^2$. Direct differentiation of the quadratic expressions gives

$$\partial_0 u + \partial_i j_i = \hbar^2 (\partial_0 z) \mathcal{E}[z], \quad \partial_0 j_i + \partial_j s_{ij} = -\hbar^2 (\partial_i z) \mathcal{E}[z].$$

For complex ζ , add its two real components. Both right sides vanish on the same shell equation, so

$$\partial_0 d^{i\pm} + \partial_j \left(j_j \pm \frac{1}{2} s_{ji} \right) = 0.$$

Integrating over a finite region gives its directional change as minus the outward flux $\int_{\partial\Omega} (j_j \pm s_{ji}/2) n_j dS$. With periodic or decaying boundaries, Parseval gives

$$\begin{aligned} \int u d^3x &= \frac{1}{2} \langle \zeta, (\widehat{M}_+^2 + \widehat{\mathbf{p}}^2 + p_f^2) \zeta \rangle = \langle \psi, \widehat{M}_+ \psi \rangle, \\ \int j_i d^3x &= \text{Re} \langle \widehat{M}_+ \zeta, \widehat{p}_i \zeta \rangle = \langle \psi, \widehat{p}_i \psi \rangle. \end{aligned}$$

Convex mixtures preserve these positivity and balance statements. The construction retains spatial phase while supplying the correct integrated readings.

Its gradient stresses cannot be discarded. With $p_f = \hbar = 1$, take equal Fourier weights at $p_x = 0, \sqrt{3}$, in periodic volume $\mathcal{V}$:

$$\psi = \frac{e^{-ix^0} + e^{i\sqrt{3}x - 2ix^0}}{\sqrt{2\mathcal{V}}}, \quad \zeta = \frac{e^{-ix^0} + 2^{-1/2} e^{i\sqrt{3}x - 2ix^0}}{\sqrt{2\mathcal{V}}}.$$

At opposite interference phase, substitution gives

$$\mathcal{V}_u = \frac{3}{2} - \frac{3\sqrt{2}}{4} > 0, \quad \mathcal{V}_{j_1} = \frac{\sqrt{3}}{2} - \frac{\sqrt{6}}{4}, \quad \mathcal{V}_{s_{22}} = -\frac{\sqrt{2}}{4} < 0.$$

No nonnegative kinetic sum of p_2^2/M can produce that transverse stress. Positive opposed densities therefore do not imply a positive kinetic representation of the entire local tensor. The source has coherent gradient structure beyond its free diagonal.

Why the diagonal still does not select the source

A conserved improvement already demonstrates the freedom:

$$\mathcal{V}_\xi^{AB} = \mathcal{V}_0^{AB} + \xi \frac{q^A q^B - \eta^{AB} q_C q^C}{\sqrt{MM'}}.$$

Its diagonal vanishes and its contraction with q_A is zero. Thus it leaves flat free propagation and integrated core/translation unchanged under suitable boundaries. It arises consistently by adding $-\xi \hbar^2 R |\zeta|^2/2$ to the same scalar action, not by changing the source alone. The curved equation acquires $\xi \hbar^2 R \zeta$, while the source acquires

$$\Delta J^{\mu\nu} = \xi \hbar^2 \left[G^{\mu\nu} |\zeta|^2 + (g^{\mu\nu} \square_g - \nabla^\mu \nabla^\nu) |\zeta|^2 \right].$$

Here R and $G^{\mu\nu}$ are the scalar curvature and Einstein tensor of the chosen metric control. In flat space $\Delta J^{00} = -\xi \hbar^2 \nabla^2 |\zeta|^2$, whose integral is a boundary term. The free diagonal kernel cannot distinguish these models. The positive construction above uses $\xi = 0$; imposing a stronger all-states pointwise selection condition or a finite-resolution instrument is a further physical requirement, not an editorial choice.

A separate positive-branch spinor source counterexample

Now change models explicitly. For the minimally coupled free Hermitian spinor action,

$$\mathcal{A}_D = \int dV \left[\frac{i\hbar}{2} \bar{\Psi} \gamma^\mu \vec{D}_\mu \Psi - p_f \bar{\Psi} \Psi \right],$$

the on-shell symmetric source in our Clifford convention is

$$J_D^{\mu\nu} = -\frac{i\hbar}{4} \bar{\Psi} \left(\gamma^\mu \vec{D}^\nu + \gamma^\nu \vec{D}^\mu \right) \Psi.$$

Raised derivatives use Gravity's $(-+++)$ signature. In a flat positive plane wave the minus sign makes $J_D^{00} = M \Psi^\dagger \Psi$, as required. This source is not the scalar allocation just constructed.

Take $p_f = \hbar = c = 1$, a periodic longitudinal length $\mathcal{L} = 2\pi/(4\sqrt{3})$ and uniform transverse mode, total volume $\mathcal{V}$. Two normalized positive modes in one polarization block are

$$p_1 = 0, \quad M_1 = 1, \quad u_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad p_2 = 4\sqrt{3}, \quad M_2 = 7, \quad u_2 = \begin{pmatrix} 2/\sqrt{7} \\ \sqrt{3/7} \end{pmatrix}.$$

Direct application of $\sigma_z + p_j \sigma_x$ gives $M_j u_j$ for both. Form the normalized state

$$\Psi(t, x) = \frac{1}{\sqrt{\mathcal{V}}} \left[\frac{2}{\sqrt{5}} u_1 e^{-it} - \frac{1}{\sqrt{5}} u_2 e^{i4\sqrt{3}x - i7t} \right].$$

Orthogonal Fourier modes ensure the integrated norm is one. At $t = x = 0$, the symmetric source is $J_D^{00} = \text{Re}(\Psi^\dagger D \Psi)$. Using $u_1^\dagger u_2 = 2/\sqrt{7}$, its diagonal contributions are $4/5 + 7/5$ and its cross contribution is $(-2/5)(1 + 7)(2/\sqrt{7})$. Therefore

$$\boxed{\mathcal{V} J_D^{00} = \frac{11}{5} - \frac{32}{5\sqrt{7}} \simeq -0.218973.}$$

Yet

$$\mathcal{V}\Psi^\dagger\Psi = 1 - \frac{8}{5\sqrt{7}} > 0, \quad M_{\text{tot}} = \frac{11}{5}, \quad P_{\text{tot}} = \frac{4\sqrt{3}}{5}, \quad M_{\text{tot}} > |P_{\text{tot}}|.$$

The negative patch is coherent interference between two positive matter modes. It is not an antiparticle or negative-probability state. A six-component lift cannot repair it: $J_D^{00} = \sum_{A,B} J_D^{AB}/36$, and local opposed source densities whose sum is $2J_D^{00}$ cannot both be nonnegative there. This disproves unsmeared classical-positive interpretation of **this** source, not every possible operational allocation.

A complete finite reciprocal comparator

One can test reciprocal coherent interaction without pretending to have completed either continuum action. In units $\hbar = c = p_{f,A} = p_{f,B} = 1$, retain momentum modes $n = -3, \dots, 3$ for each of two carriers on a circle of circumference 2π . Give each carrier a degenerate two-state internal space. The total Hilbert dimension is $7^2 \times 4 = 196$.

Let $T|n\rangle = |n+1\rangle$ inside the retained range and zero at its upper edge. Define the compressed relative-position operator

$$F = I + \frac{1}{2}(T \otimes T^\dagger + T^\dagger \otimes T),$$

which is the orthogonal compression of multiplication by $1 + \cos(x_A - x_B)$ and is therefore positive. Set

$$\begin{aligned} \widehat{M}_0 &= \sqrt{1 + P_A^2} + \sqrt{1 + P_B^2}, \\ V &= gF \otimes (I + \sigma_x^A \sigma_x^B), \quad g \geq 0, \\ \widehat{M}_{\text{tot}} &= \widehat{M}_0 + V. \end{aligned}$$

Both factors in V are positive. Every off-diagonal momentum shift is opposite on the two carriers, including after compression; hence $[\widehat{M}_{\text{tot}}, P_A + P_B] = 0$. The autonomous generator conserves itself, while

$$\frac{dP_A}{dx^0} = i[\widehat{M}_{\text{tot}}, P_A], \quad \frac{dP_B}{dx^0} = i[\widehat{M}_{\text{tot}}, P_B] = -\frac{dP_A}{dx^0}.$$

The interaction is counted once in the complete core. Its positivity and the diagonal free inequality prove

$$\widehat{M}_{\text{tot}} \geq \widehat{M}_0 \geq |P_A + P_B|.$$

Consequently $\widehat{Q}_k^\pm = \widehat{M}_{\text{tot}} \pm k(P_A + P_B)/2$ are positive and conserved for every directional projection $|k| \leq 1$. This is a global finite-system statement, not a local tensor source.

For $g = 0.25$, take normalized $v_n \propto e^{-n^2/2}$ for each carrier and internal state $|00\rangle$. Evolving with $e^{-i\widehat{M}_{\text{tot}}}$ for one unit of x^0 and tracing out both momentum spaces gives internal negativity 0.3395840322, where negativity is $(\|\rho_{\text{int}}^{T_B}\|_1 - 1)/2$. With displaced initial amplitudes $v_n e^{-in/2}$ and $v_n e^{in/2}$, the mean impulses are $+0.1190235253$ and -0.1190235253 . The reproduced finite calculation conserves norm and complete core to below 10^{-12} and 10^{-11} respectively; the exact total-momentum commutator vanishes.

This demonstrates that entanglement, reciprocal momentum transfer and positive conserved global directional charges can coexist. The finite cutoff is spatially nonlocal and frame specific, and the interaction is not a derived attractive gravitational coupling. No causal continuum, local source tensor or curved completion is inferred from it.

Recurrence, gravitational sourcing and the remaining boundary

The kinetic contraction $-\eta_{AB}j^{AB} = p_f^2/M$ is the free recurrence quantity. The weak temporal deformation responds instead to core **plus** transport. In the aligned frame define the selector

$$\mathcal{W}_{i\sigma,j\tau} = \frac{1}{36} + \sigma\tau\delta_{ij}.$$

Since $N \simeq 1 - \theta^{00} - \sum_i \theta^{ii}$,

$$\delta N = -\mathcal{W}_{AB}\delta\theta^{AB}, \quad \mathcal{W}_{AB}J^{AB} = \mathcal{M} + \text{tr } \mathcal{C}, \quad -\eta_{AB}J^{AB} = \mathcal{M} - \text{tr } \mathcal{C}.$$

A null kinetic carrier has zero recurrence contraction but temporal projection $2M$. It still sources gravity. The indexed L lift changes display, whereas $J_{\text{coord}} = \phi^{-1}J_{\text{local}}\phi^{-\text{T}}$ changes frame. Neither operation selects an extra force. For a localized stationary complete source with vanishing boundary terms, $\partial_k J^{kj} = 0$ implies $\int J^{ij}d^3x = \int \partial_k(x^i J^{kj})d^3x = 0$. Internal stresses without their supports therefore cannot be used to infer an extra independent long-range stress monopole.

A curved-region balance also needs its comparison field. For a conserved symmetric source and covector K_ν ,

$$\nabla_\mu(J^{\mu\nu}K_\nu) = J^{\mu\nu}\nabla_{(\mu}K_{\nu)}.$$

Four arbitrary observer components are not four conserved Killing charges on a generic curved Stage. Boundary and interaction/Stage exchange must be kept in a common accounting.

Finally, this one-carrier scalar positivity result is not a field-vacuum theorem. A field state may contain pair coherence absent from its one-body kernel. A positive operator with zero vacuum expectation would annihilate the vacuum and could not retain nonzero vacuum–pair matrix elements. Renormalized source mean, connected source noise and causal detector response therefore remain distinct; this distinction is central in Hu and Verdaguer’s account of stochastic gravity. Real unbound momentum can source gravity without forming a stable particle, but a noise variance is not automatically extra positive mean core.

Proposed §4.8 receives a constructive scalar allocation, a precise spinor-source obstruction and a bounded reciprocal interaction example. The native task left by them is concrete: select one coherent interaction whose response, complete source and operational positive accounting follow together, with its material and Stage state laws specified.

Derivation 5.2A — Botic-slot stage dressing and the chi-squared group velocity

Purpose

This appendix supports Chapter 5.2. It separates conserved spatial momentum content from the generator that converts that content into phase evolution and translation, then derives the massive translation law

$$\dot{x}_k = \chi^2 c \frac{p_k}{M_\chi}.$$

The result is a generator theorem within a stated two-slot dressing architecture. It does not determine the history $\chi(t)$, amend the ADMC ledger, or decide which dressed quantity gravitates.

Role, regime, premises, and status

- **Role:** derivation of the homogeneous stage-dressed generator and its group velocity.
- **Regime:** a free narrow wave packet on a spatially homogeneous, slowly varying stage; no local gravitational field.
- **Premises:** the Chapter 4 phase law, the Dirac-form core-momentum operator, spatial translation symmetry, and a multiplicative dressing of its fermic and botic slots.

Notation ledger

- p_f — fixed fermic momentum scale.
- $\hat{\mathbf{p}}$ — spatial momentum operator; $\mathbf{p}$ is its wave-packet label and $p = |\mathbf{p}|$.
- β, α_i — Dirac matrices satisfying $\beta^2 = 1$, $\{\alpha_i, \alpha_j\} = 2\delta_{ij}$, and $\{\beta, \alpha_i\} = 0$.
- $\chi(t) > 0$ — normalized homogeneous translation yield, with $\chi = 1$ at the reference inertial epoch.
- $\hat{M}_\chi$ — stage-dressed momentum-unit generator.

- M_χ — positive instantaneous eigenvalue or principal-symbol magnitude of $\hat{M}_\chi$.
- $\hat{H}_\chi = c\hat{M}_\chi$ — energy-unit generator.
- x_k, p_k — coordinate and conserved momentum label along a selected direction.

If an unnormalized yield variable is used, every χ in this appendix means the ratio $\chi(t)/\chi_o$. The conventional choice $\chi_o = 1$ suppresses that distinction.

Flat-space degeneracy

In the inertial theory, the same operator appears in two roles:

$$\hat{M} = \beta p_f + \alpha \cdot \hat{\mathbf{p}}.$$

Its labels identify the carrier content $(p_f, \mathbf{p})$, while its energy-unit form generates phase evolution,

$$i\hbar\partial_t\Psi = c\hat{M}\Psi.$$

For a free positive-branch eigenstate, the Dirac anticommutation relations give

$$\hat{M}^2 = p_f^2 + \hat{\mathbf{p}}^2,$$

and hence

$$M = \sqrt{p_f^2 + p^2}.$$

Flat space makes content and generator numerically degenerate: the conserved momentum labels also determine the time-independent generator eigenvalue. A time-dependent stage separates those roles. Spatial homogeneity can preserve $\mathbf{p}$ even while the generator built from it changes.

The two-slot multiplicative ansatz

The most economical stage dressing preserves the first-order operator structure and assigns one homogeneous factor to each slot:

$$\hat{M}[Y_f, Y_b] = Y_f(t) \beta p_f + Y_b(t) \alpha \cdot \hat{\mathbf{p}}.$$

The factors are dimensionless and recover the inertial operator when $Y_f = Y_b = 1$. A common homogeneous factor changes the normalization of the complete phase generator. After the reference time normalization is fixed, the relative slot dressing is the nontrivial quantity. The expansion branch used here fixes the fermic reference scale and writes

$$Y_f = 1, \quad Y_b = \chi(t).$$

Thus

$$\boxed{\hat{M}_\chi = \beta p_f + \chi(t) \alpha \cdot \hat{\mathbf{p}}}.$$

Because χ depends on time but not position,

$$[\hat{H}_\chi, \hat{p}_k] = 0.$$

Equivalently, the Hamiltonian has no x_k dependence, so Hamilton's equation gives

$$\dot{p}_k = -\frac{\partial H_\chi}{\partial x_k} = 0.$$

The spatial momentum label is therefore conserved by homogeneous translation symmetry. This is the conserved content used below. It is not the same object as the time-dependent eigenvalue of the generator.

Instantaneous eigenvalue

Squaring the dressed operator gives

$$\begin{aligned}\hat{M}_\chi^2 &= (\beta p_f + \chi \alpha_i \hat{p}_i) (\beta p_f + \chi \alpha_j \hat{p}_j) \\ &= p_f^2 + \chi p_f (\beta \alpha_j + \alpha_j \beta) \hat{p}_j + \chi^2 \alpha_i \alpha_j \hat{p}_i \hat{p}_j.\end{aligned}$$

The mixed term vanishes because $\{\beta, \alpha_j\} = 0$. The antisymmetric part of $\alpha_i \alpha_j$ vanishes against the symmetric product $\hat{p}_i \hat{p}_j$, leaving

$$\hat{M}_\chi^2 = p_f^2 + \chi^2 \hat{\mathbf{p}}^2.$$

On the positive branch,

$$\boxed{M_\chi = \sqrt{p_f^2 + \chi^2 p^2}}.$$

The labels p_f and $\mathbf{p}$ remain fixed in the stated regime, but M_χ changes when $\chi(t)$ changes. The dressed phase frequency is

$$\omega_\chi = \frac{c M_\chi}{\hbar}.$$

Group-velocity theorem

For a narrow packet whose instantaneous dispersion is

$$H_\chi(\mathbf{p}, t) = c \sqrt{p_f^2 + \chi^2 p^2},$$

the center velocity is the momentum derivative of the energy-unit generator. Along direction k ,

$$\begin{aligned}\dot{x}_k &= \frac{\partial H_\chi}{\partial p_k} \\ &= c \frac{1}{2} (p_f^2 + \chi^2 p^2)^{-1/2} (2\chi^2 p_k) \\ &= \chi^2 c \frac{p_k}{M_\chi}.\end{aligned}$$

Therefore

$$\dot{x}_k = \frac{\partial(cM_\chi)}{\partial p_k} = \chi^2 c \frac{p_k}{M_\chi}.$$

The two powers of χ have one algebraic origin: the bosic slot enters the squared eigenvalue as $\chi^2 p^2$, and differentiation is taken with respect to the conserved momentum coordinate p_k .

Placement-A diagnostic and whole-generator contrast

The earlier Chapter 5 map was

$$\dot{x}_k^{(A)} = \chi c \frac{p_k}{M}, \quad M = \sqrt{p_f^2 + p^2}.$$

That velocity does have a generating Hamiltonian:

$$H_A = c\chi(t)M, \quad \frac{\partial H_A}{\partial p_k} = \chi c \frac{p_k}{M}.$$

The problem is not the literal absence of a generator. The problem is that H_A dresses the whole generator,

$$\hat{M}_A = \chi (\beta p_f + \alpha \cdot \hat{\mathbf{p}}),$$

whereas the published map was presented as a downstream multiplier while the phase generator remained undressed. Those two statements cannot both represent the packet dynamics.

Whole-generator dressing also co-scales the fermic and bosic slots. On the matched-standard branch, that common rescaling is absorbed into the common clock/phase normalization and supplies no nontrivial photon-to-atomic endpoint ratio. Bosic-slot dressing instead leaves the fermic reference scale fixed while the spatial slot changes. This produces the required massive history and the lightlike endpoint asymmetry developed in Derivations 5.3A and 5.4A.

Within the two-slot multiplicative ansatz, the scope of the placement statement can be made exact. Consider

$$\hat{M}_{a,b} = \chi^a \beta p_f + \chi^b \alpha \cdot \hat{\mathbf{p}}.$$

The required lightlike law $\dot{x} = \chi c$ fixes $b = 1$. Holding the intrinsic fermic scale fixed fixes $a = 0$. These conditions select

$$(a, b) = (0, 1),$$

which is the basic-slot operator $\hat{M}_\chi$. This is uniqueness within this ansatz and these requirements, not uniqueness over every possible Hamiltonian, nonmultiplicative coupling, or stage theory.

Limiting checks

Inertial limit

At $\chi = 1$,

$$M_\chi \rightarrow M, \quad \dot{x}_k \rightarrow c \frac{p_k}{M}.$$

The flat operator and inertial translation theorem are recovered.

Zero directed momentum

For $p_k = 0$,

$$\dot{x}_k = 0.$$

Reversing the selected orientation changes the signs of p_k and $\dot{x}_k$ but not M_χ .

Nonrelativistic limit

For $\chi p \ll p_f$,

$$M_\chi = p_f + \frac{\chi^2 p^2}{2p_f} + O\left(\frac{\chi^4 p^4}{p_f^3}\right),$$

so

$$\dot{x}_k \simeq \chi^2 c \frac{p_k}{p_f}.$$

With $p_f = m_0 c$, the excess energy-unit generator is $\chi^2 p^2 / (2m_0)$.

Lightlike limit

For $p_f = 0$,

$$M_\chi = \chi|p|,$$

and therefore

$$\dot{x}_k = \chi c \frac{p_k}{|p|}.$$

Forward lightlike propagation gives $\dot{x} = \chi c$. The corrected massive placement therefore preserves the Chapter 5 lightlike law.

Result and boundaries

This appendix establishes, within a homogeneous two-slot multiplicative dressing architecture:

1. spatial translation symmetry conserves $\mathbf{p}$;
2. the basic-slot generator has positive eigenvalue $M_\chi = \sqrt{p_f^2 + \chi^2 p^2}$;
3. its group velocity is $\dot{x}_k = \chi^2 c p_k / M_\chi$;
4. the inertial and lightlike limits are correct;
5. the old χ map is the group velocity of a whole-generator dressing, not of the basic-slot generator;
6. the placement is unique only within the stated ansatz and requirements.

The derivation does **not** amend ADMC. It keeps conserved spatial content, the dressed generator, and gravitational source weight as three distinct ledger questions. In particular, it does not assume that the ADMC even entry is M_χ , and it does not determine whether a cosmological source law should use undressed content, dressed generator weight, or a stage-plus-carrier constraint.

Chapter 5.2 may cite this appendix for the operator, eigenvalue, placement comparison, and χ^2 theorem. Chapter 5.3 supplies the free-carrier correspondence test; Chapter 5.4 develops the lightlike endpoint package.

Derivation 5.3A — Free-carrier correspondence under a yield history

Purpose

This appendix supports Chapter 5.3. It converts the basic-slot group velocity into the clean bound frame and proves that a free massive M1 carrier follows the exact special-relativistic FLRW peculiar-velocity law when its conserved spatial content is identified with comoving momentum.

The theorem is conditional on a prescribed homogeneous yield history. It checks the kinematics produced by that history; it does not derive $H(t)$ or a Friedmann equation.

Role, regime, premises, and status

- **Role:** exact kinematic correspondence theorem and placement diagnostic.
- **Regime:** free carrier, homogeneous yield history, no local gravitational field, clean bound-frame branch.
- **Premises:** Derivation 5.2A, normalized yield, co-scaling rods, matched clocks, and subtraction of the common background recession term when peculiar velocity is defined.

Notation ledger

- $\chi(t) > 0$ — normalized yield entering the dressed generator.
- χ_e, χ_o — unnormalized endpoint values when a ratio must be displayed.
- $X \equiv \chi/\chi_o$ — normalized yield ratio if $\chi_o \neq 1$.
- $a_B \equiv \chi_o/\chi = 1/X$ — clean-branch bound-frame scale factor.
- $H \equiv \dot{a}_B/a_B = -\dot{X}/X$ — prescribed fractional history rate.
- $p_f = m_0 c$ — fixed fermic momentum scale.
- $\mathbf{p}$ — conserved M1 spatial momentum content; $p = |\mathbf{p}|$.
- $M_X \equiv \sqrt{p_f^2 + X^2 p^2}$ — normalized dressed generator magnitude.
- v_B — peculiar velocity measured in the bound frame.
- q — standard FLRW comoving momentum used only in the correspondence comparison.

For compactness, the derivation first keeps $X = \chi/\chi_o$ explicit. Setting $\chi_o = 1$ gives $X = \chi$ and recovers the notation of Derivation 5.2A.

Clean bound-frame conversion

On the clean branch, local true-frame rods scale with the normalized yield:

$$\ell_T = \ell_o X.$$

A true-frame coordinate interval is therefore reported in bound-frame length units as

$$x_B = \frac{x_T}{X} = a_B x_T, \quad a_B = \frac{1}{X} = \frac{\chi_o}{\chi}.$$

Differentiating $x_B = a_B x_T$ gives a background term $\dot{a}_B x_T$ and a local-motion term $a_B \dot{x}_T$. Peculiar velocity is the local-motion term after the common recession of the bound-frame coordinate grid is subtracted:

$$\boxed{v_B = a_B \dot{x}_T}.$$

The clean branch uses matched clocks, so no additional time conversion appears.

Bosic-slot free-carrier velocity

With the normalized yield X , Derivation 5.2A gives

$$M_X = \sqrt{p_f^2 + X^2 p^2},$$

and, along the direction of motion,

$$\dot{x}_T = X^2 c \frac{p}{M_X}.$$

The bound-frame peculiar velocity is therefore

$$\begin{aligned}
v_B &= a_B X^2 c \frac{p}{M_X} \\
&= X c \frac{p}{\sqrt{p_f^2 + X^2 p^2}} \\
&= c \frac{p/a_B}{\sqrt{p_f^2 + (p/a_B)^2}}.
\end{aligned}$$

Thus

$$\boxed{v_B = c \frac{p/a_B}{\sqrt{p_f^2 + (p/a_B)^2}}}.$$

The numerator contains the bound-frame momentum expression p/a_B , while the M1 content label p itself remains conserved.

Exact FLRW correspondence

For a freely moving particle in a spatially flat FLRW description, spatial homogeneity conserves comoving momentum q . The physical momentum measured by a comoving observer is q/a , and special-relativistic velocity is

$$v_{\text{FLRW}} = c \frac{q/a}{\sqrt{m_0^2 c^2 + (q/a)^2}}.$$

Using

$$p_f = m_0 c, \quad a = a_B, \quad \boxed{q \equiv p},$$

gives

$$v_{\text{FLRW}} = v_B$$

at every speed. M1's conserved spatial content is therefore the variable that standard FLRW kinematics calls comoving momentum in this homogeneous correspondence regime.

This equality is an overlap theorem. It demonstrates that the M1 stage-dressing variables reproduce known free-particle kinematics under a prescribed history. It is not an independent observational prediction and does not explain why the history has any particular form.

Exact drag law

Define the bound-frame expressed momentum

$$y \equiv \frac{p}{a_B} = Xp.$$

Since p is conserved,

$$\dot{y} = -\frac{\dot{a}_B}{a_B}y = -Hy.$$

Write the velocity as

$$v_B(y) = c \frac{y}{\sqrt{p_f^2 + y^2}}.$$

Its derivative is

$$\frac{dv_B}{dy} = c \frac{p_f^2}{(p_f^2 + y^2)^{3/2}}.$$

Therefore

$$\dot{v}_B = -Hc \frac{yp_f^2}{(p_f^2 + y^2)^{3/2}}.$$

The velocity itself satisfies

$$1 - \frac{v_B^2}{c^2} = \frac{p_f^2}{p_f^2 + y^2},$$

so the previous expression becomes

$$\boxed{\dot{v}_B = -Hv_B \left(1 - \frac{v_B^2}{c^2}\right)}.$$

This is the exact relativistic FLRW geodesic drag law in velocity form. The history enters only through the supplied function $H(t) = -\dot{X}/X$.

Limiting checks

Inertial history

If $X = 1$ is constant, then $a_B = 1$, $H = 0$, and

$$v_B = c \frac{p}{\sqrt{p_f^2 + p^2}}, \quad \dot{v}_B = 0.$$

The inertial free-particle result is recovered.

Nonrelativistic massive carrier

For $p/a_B \ll p_f$,

$$v_B \simeq c \frac{p}{a_B p_f} = \frac{p}{m_0 a_B}.$$

Hence

$$v_B \propto a_B^{-1}, \quad \dot{v}_B \simeq -H v_B.$$

The standard nonrelativistic peculiar-momentum decay is recovered.

Lightlike carrier

For $p_f = 0$,

$$v_B = c$$

for forward propagation, independent of a_B . The true-frame speed is still $\dot{x}_T = Xc$, while the bound-frame rod conversion supplies the compensating factor $a_B = 1/X$.

Diagnostic for the old placement

The old Chapter 5 map used

$$\dot{x}_T^{(A)} = Xc \frac{p}{M}, \quad M = \sqrt{p_f^2 + p^2}.$$

Its bound-frame peculiar velocity is

$$\begin{aligned} v_B^{(A)} &= a_B \dot{x}_T^{(A)} \\ &= \frac{1}{X} Xc \frac{p}{M} \\ &= c \frac{p}{\sqrt{p_f^2 + p^2}}. \end{aligned}$$

Therefore

$$\boxed{\dot{v}_B^{(A)} = 0}$$

for every freely moving massive carrier, regardless of $H(t)$. Placement A agrees with the photon limit because $v_B = c$ there, but it fails the massive FLRW overlap already in the nonrelativistic limit.

As shown in Derivation 5.2A, this old velocity can be generated by the whole-generator Hamiltonian $H_A = cXM$. Its failure here is empirical and structural: whole-generator dressing plus the clean bound-frame conversion gives constant massive peculiar velocity.

Result and boundaries

Given

$$a_B = \frac{\chi_o}{\chi}, \quad \hat{M}_X = \beta p_f + X\alpha \cdot \hat{\mathbf{p}},$$

the basic-slot group velocity yields

$$v_B = c \frac{p/a_B}{\sqrt{p_f^2 + (p/a_B)^2}}$$

and

$$\dot{v}_B = -Hv_B \left(1 - \frac{v_B^2}{c^2} \right).$$

These are exactly the relativistic FLRW free-carrier relations under $p \equiv q$. The result establishes the homogeneous kinematic overlap and excludes the old placement on the same clean branch.

It does not derive $a_B(t)$, $H(t)$, a Friedmann constraint, a gravitational source law, structure growth, or any observational fit. Chapter 5.3 may cite it only as a free-carrier correspondence under a prescribed yield history.

Derivation 5.4A — Lightlike propagation and matched endpoint standards

Purpose

This appendix supports Chapter 5.4. It derives the lightlike propagation kernel, proves the neighboring-feature endpoint relation, and states the rod and clock assumptions needed to read that relation as wavelength redshift and duration stretching.

The derivation keeps three objects distinct: the conserved external spatial wavenumber, the drifting generator frequency, and the wavelength reported relative to local rods.

Role, regime, premises, and status

- **Role:** lightlike kinematics plus a conditional matched-endpoint measurement dictionary.
- **Regime:** homogeneous yield history, geometric propagation, neighboring signal features, negligible peculiar motion and endpoint evolution during one feature separation.
- **Premises:** basic-slot dressing, spatial homogeneity, the clean bound-frame branch, co-scaling rods, and matched endpoint clocks.

Notation ledger

- $\chi(t) > 0$ — normalized yield entering the generator; $\chi_o = 1$ in the compact formulas.
- χ_e, χ_o — endpoint yield values when normalization is restored.
- p_f — fermic momentum scale; $p_f = 0$ for a lightlike carrier.
- $\mathbf{k}$ — external spatial wavevector on the homogeneous true-frame chart.
- $\mathbf{p} = \hbar \mathbf{k}$ — conserved spatial momentum content.
- ω_χ — instantaneous generator angular frequency.
- x_T, t — true-frame propagation coordinate and common history parameter.
- $\delta t_e, \delta t_o$ — neighboring-feature endpoint separations in t .
- $\Delta t_e, \Delta t_o$ — durations reported by local endpoint clocks.
- ℓ_T — local true-frame rod standard.
- $a_B = \chi_o / \chi$ — clean-branch bound-frame scale factor.

- η — conformal propagation time.

Whenever the absolute normalization is not $\chi_o = 1$, replace the generator's χ by $X = \chi/\chi_o$. Endpoint observables depend only on χ_e/χ_o .

Lightlike dispersion and group velocity

The basic-slot generator magnitude is

$$M_\chi = \sqrt{p_f^2 + \chi^2 p^2}.$$

For a lightlike carrier, $p_f = 0$, so

$$\boxed{M_\chi = \chi|p|}.$$

The energy-unit generator and angular frequency are

$$H_\chi = cM_\chi = c\chi|p|, \quad \omega_\chi = \frac{H_\chi}{\hbar} = \chi c|\mathbf{k}|.$$

The group velocity is

$$\dot{x}_k = \frac{\partial H_\chi}{\partial p_k} = \chi c \frac{p_k}{|p|}.$$

For forward propagation,

$$\boxed{\dot{x}_T = \chi c}.$$

Thus the χ^2 massive theorem reduces to the original χ lightlike law because the dressed magnitude itself contributes one factor of χ in the denominator.

Conserved external wavenumber and drifting frequency

The generator is homogeneous in space:

$$\frac{\partial H_\chi}{\partial x_k} = 0.$$

Hamilton's equation therefore gives

$$\dot{p}_k = 0,$$

and, with fixed $\hbar$,

$$\boxed{\dot{k}_k = 0}.$$

The external spatial phase gradient and true-frame coordinate wavelength

$$\lambda_T = \frac{2\pi}{|\mathbf{k}|}$$

remain fixed in flight. The generator frequency does not:

$$\omega_\chi(t) = \chi(t)c|\mathbf{k}|, \quad \frac{\dot{\omega}_\chi}{\omega_\chi} = \frac{\dot{\chi}}{\chi}.$$

The signal therefore undergoes generator drift at fixed external spatial content. This is not an in-flight stretching of the external wavelength. A changing wavelength is reported only after the fixed coordinate pattern is compared with endpoint rods whose true-frame size depends on χ .

Propagation kernel

Integrating the forward lightlike velocity from emission to observation gives the true-frame path interval

$$\boxed{\mathcal{D}_\chi(t_e, t_o) = c \int_{t_e}^{t_o} \chi(t) dt}.$$

The integral is a propagation kernel. It is not by itself a luminosity distance, angular-diameter distance, ruler distance, or flux law.

On the clean branch,

$$a_B = \frac{1}{\chi}$$

in the normalized convention. Define conformal time by

$$d\eta \equiv \frac{dt}{a_B}.$$

Then

$$\boxed{d\eta = \chi dt}$$

and

$$\mathcal{D}_\chi = c \int_{\eta_e}^{\eta_o} d\eta.$$

With an unnormalized yield, the exact identity is

$$d\eta = \frac{\chi}{\chi_o} dt;$$

the conventional $\chi_o = 1$ choice gives the boxed form. The lightlike kernel is therefore the standard conformal propagation kernel under the clean correspondence. It does not by itself change the usual horizon accounting.

Neighboring-feature equal-path derivation

Consider two neighboring signal features emitted by the same source and received by the same observer. Assume that, over the small endpoint separations, source and observer keep the same true-frame coordinate separation and that changes in peculiar motion, lensing, and inhomogeneity are negligible.

The first feature obeys

$$D_T = c \int_{t_e}^{t_o} \chi(t) dt.$$

The second feature follows the same path:

$$D_T = c \int_{t_e + \delta t_e}^{t_o + \delta t_o} \chi(t) dt.$$

Subtracting gives

$$0 = c \int_{t_o}^{t_o + \delta t_o} \chi(t) dt - c \int_{t_e}^{t_e + \delta t_e} \chi(t) dt.$$

For neighboring features, χ varies negligibly across each endpoint interval. To first order,

$$0 = c\chi_o \delta t_o - c\chi_e \delta t_e.$$

Hence

$$\boxed{\chi_o \delta t_o = \chi_e \delta t_e}$$

and

$$\boxed{\frac{\delta t_o}{\delta t_e} = \frac{\chi_e}{\chi_o}}.$$

This equation follows from the common path and the lightlike propagation law. Turning it into an observed duration relation requires the clock assumption below.

Matched endpoint rods

On the clean branch, local true-frame rods scale as

$$\ell_T(t) = \ell_o \frac{\chi(t)}{\chi_o}.$$

The conserved external wavelength $\lambda_T = 2\pi/|\mathbf{k}|$ is therefore reported in local rod units as

$$\lambda_B(t) \propto \frac{\lambda_T}{\ell_T(t)} \propto \frac{\chi_o}{\chi(t)}.$$

For the same transported phase pattern compared at emission and observation,

$$\frac{\lambda_{B,o}}{\lambda_{B,e}} = \frac{\chi_e}{\chi_o}.$$

Thus the measured wavelength relation is

$$\boxed{1 + z \equiv \frac{\lambda_o}{\lambda_e} = \frac{\chi_e}{\chi_o}}.$$

The same ratio appears in frequency language. An emitted feature with local angular frequency ω_e fixes an external wavevector through

$$\omega_e = \frac{\chi_e}{\chi_o} c |\mathbf{k}|.$$

Spatial homogeneity preserves $\mathbf{k}$ in flight, so the observation-epoch generator frequency is

$$\omega_o = c |\mathbf{k}| = \frac{\chi_o}{\chi_e} \omega_e.$$

Therefore

$$\frac{\omega_e}{\omega_o} = \frac{\chi_e}{\chi_o} = 1 + z.$$

This frequency statement assumes that emitter and observer use matched local spectral standards. Derivation 5.6A supplies a hydrogenic proxy in which those standards remain fixed while rods co-scale.

Matched endpoint clocks and duration stretching

The clean branch takes local tick frequency to be fixed relative to the common history parameter:

$$\nu_T \propto \chi^0.$$

Reported endpoint durations are therefore proportional to the corresponding background intervals with the same conversion factor at emission and observation:

$$\frac{\Delta t_o}{\Delta t_e} = \frac{\delta t_o}{\delta t_e}.$$

Using the equal-path result,

$$\boxed{\frac{\Delta t_o}{\Delta t_e} = \frac{\chi_e}{\chi_o} = 1 + z}.$$

Equivalently,

$$\boxed{\Delta t_o = (1 + z)\Delta t_e}.$$

One endpoint ratio therefore governs wavelength redshift and neighboring-feature duration stretching, but only after the co-scaling rod and matched-clock assumptions are applied.

Limiting checks

Inertial history

If χ is constant, then

$$\omega_\chi = \text{constant}, \quad \delta t_o = \delta t_e, \quad z = 0.$$

Normalization invariance

Under a constant rescaling $\chi \mapsto C\chi$, the normalized generator uses $X = \chi/\chi_o$, while endpoint observables use

$$\frac{C\chi_e}{C\chi_o} = \frac{\chi_e}{\chi_o}.$$

No measured relation depends on the arbitrary normalization.

Direction reversal

Reversing the propagation orientation changes the sign of $\dot{x}_T$ but not M_χ , ω_χ , or the endpoint ratios.

Result and boundaries

This appendix establishes:

1. $p_f = 0$ implies $M_\chi = \chi|p|$ and $\dot{x}_T = \chi c$;
2. spatial homogeneity conserves external $\mathbf{k}$ while $\omega_\chi = \chi c|\mathbf{k}|$ drifts;
3. the path kernel is $c \int \chi dt$;
4. neighboring equal paths give $\chi_o \delta t_o = \chi_e \delta t_e$;
5. co-scaling rods give $1 + z = \chi_e/\chi_o$;
6. matched clocks give $\Delta t_o/\Delta t_e = 1 + z$;
7. $d\eta = \chi dt$ in the normalized clean convention.

It does not derive luminosity distance, angular-diameter distance, flux or photon-number transport, lensing, full radiative transfer, source evolution, blackbody evolution, recombination, thermal history, or Friedmann dynamics. It also does not claim that the external wavelength stretches during flight.

Chapter 5.4 may cite this appendix for the lightlike path, generator drift, endpoint comparison, and conditional matched-standard signal dictionary.

Derivation 5.6A — Local covariance and scoped sector realizations

Purpose

This appendix supports Chapter 5.6. It states the clean true-frame/bound-frame transform, derives its local light and ranging condition, and tests that transform in three controlled settings:

1. an adiabatic hydrogenic electromagnetic proxy;
2. an adiabatic Newtonian bound orbit;
3. linear nonrelativistic matter growth at a prescribed background history.

The calculations are realizations of a covariance target, not a proof that every interaction sector follows one universal law. Strong and weak closure remain open, and exact local indistinguishability is not established.

Role, regime, premises, and status

- **Role:** conditional local-covariance derivation and scoped sector correspondence.
- **Regime:** homogeneous slowly varying yield; clean branch; adiabatic bound systems; hydrogenic EM proxy; Newtonian/quasi-static gravity; linear nonrelativistic perturbations.
- **Premises:** basic-slot χ^2 kinetics, fixed content constants, co-scaling rods, matched clocks, $e^2 \propto \chi$ in the EM realization, and $G_{\text{eff}} \propto \chi$ in the gravity realization.

Notation ledger

- $X \equiv \chi/\chi_o$ — normalized yield, with $X_o = 1$.
- $a_B \equiv X^{-1}$ — clean-branch bound-frame scale factor.
- x_T, x_B — true-frame and bound-frame spatial coordinates.
- t — common history parameter and clean-branch clock time.
- ℓ_T — local true-frame rod length.
- $p_f = m_0 c$ — fixed fermic content scale.

- $\hbar, c, m_0$ — fixed content constants in the proxy calculations.
- $e^2(X)$ — Coulomb coupling in a convention with $\alpha_X = e^2/(\hbar c X)$.
- $G_{\text{eff}}(X)$ — effective Newtonian coupling on the true-frame chart.
- ρ_T, ρ_B — true-frame and bound-frame matter densities.
- δ — density contrast, invariant under the homogeneous volume conversion.
- $H = \dot{a}_B/a_B = -\dot{X}/X$ — prescribed background rate.

The formulas below use X to keep normalization explicit. In Chapter 5 notation with $\chi_o = 1$, replace X by χ .

Clean true-frame/bound-frame transform

The clean branch takes local true-frame rods to co-scale linearly with the normalized yield:

$$\boxed{\ell_T(X) = \ell_o X}.$$

A local observer reports a true-frame interval by dividing by that rod standard. The corresponding bound-frame coordinate is

$$\boxed{x_B = \frac{x_T}{X}},$$

and the bound-frame scale factor is

$$\boxed{a_B = \frac{1}{X} = \frac{\chi_o}{\chi}}.$$

A bound length that co-scales as $R_T = R_o X$ is therefore constant locally:

$$R_B = \frac{R_T}{X} = R_o.$$

A fixed true-frame separation instead appears as $D_B = a_B D_T$. This transform is bookkeeping between two descriptions; it does not by itself supply the sector forces that make a bound length track X .

The clean branch also takes local tick frequencies to be fixed relative to t :

$$\boxed{\nu_T(X) = \nu_o}.$$

Thus $dt_B = dt$ in the local comparisons used below.

Local light and radar-ranging condition

Lightlike motion in the true frame is

$$\dot{x}_T = cX.$$

During one local tick ν_o^{-1} , the true-frame light distance is

$$\ell_{\text{tick},T} = \frac{cX}{\nu_o}.$$

Measured in local rods,

$$\frac{\ell_{\text{tick},T}}{\ell_T} = \frac{cX/\nu_o}{\ell_o X} = \frac{c}{\nu_o \ell_o},$$

which is independent of X . Local observers therefore report a fixed light speed even though both the true-frame light distance per tick and the rod length co-scale.

Radar ranging gives the same condition. For a co-scaled local radius

$$r_T = r_o X,$$

the true-frame round-trip time is

$$\Delta t_T = \frac{2r_T}{cX} = \frac{2r_o}{c}.$$

The local tick count $\nu_o \Delta t_T$ is constant. This is the operational local-covariance condition used by the sector checks. It is not a general proof that every probe is insensitive to X .

Hydrogenic electromagnetic proxy

Premises

Use the nonrelativistic limit of the bosic-slot generator after the rest phase is removed:

$$H_{\text{kin}} = \frac{X^2 \mathbf{p}^2}{2m_e}.$$

Take

$$\hbar = \text{constant}, \quad m_e = \text{constant}, \quad e^2(X) = e_o^2 X.$$

The last relation is a scoped coupling realization, not a change in a conserved content constant. The hydrogenic proxy Hamiltonian is

$$H_H(X) = \frac{X^2 \mathbf{p}^2}{2m_e} - \frac{e_o^2 X}{r}.$$

The history is assumed adiabatic on atomic timescales, so the bound state follows the instantaneous spectrum.

Rod scale

For a state of characteristic radius r , uncertainty gives $p \sim \hbar/r$. The energy scale is therefore

$$E(r) \sim \frac{X^2 \hbar^2}{2m_e r^2} - \frac{e_o^2 X}{r}.$$

Stationarity requires

$$\frac{dE}{dr} \sim -\frac{X^2 \hbar^2}{m_e r^3} + \frac{e_o^2 X}{r^2} = 0.$$

Hence the characteristic Bohr scale is

$$a_0(X) \sim \frac{X^2 \hbar^2}{m_e e^2(X)} = X \frac{\hbar^2}{m_e e_o^2} = X a_{0,o}.$$

The atomic proxy rod co-scales exactly as required by the clean transform.

Spectral scale

The bound-state energies inherit the hydrogenic form with the dressed kinetic coefficient:

$$E_n(X) = -\frac{m_e e^4(X)}{2X^2 \hbar^2 n^2}.$$

Since $e^2(X) = e_o^2 X$, one has $e^4(X) = e_o^4 X^2$, so

$$\boxed{E_n(X) = -\frac{m_e e_o^4}{2\hbar^2 n^2}}.$$

Transition energies and frequencies are therefore independent of X :

$$\Delta E \propto X^0, \quad \nu_{\text{atom}} = \frac{\Delta E}{h} \propto X^0.$$

The proxy supplies the matched-clock condition used in Derivation 5.4A.

Fixed fine-structure and velocity checks

The operative dimensionless coupling is

$$\alpha_X \equiv \frac{e^2(X)}{\hbar c X}.$$

With $e^2(X) = e_o^2 X$,

$$\boxed{\alpha_X = \frac{e_o^2}{\hbar c} = \text{constant}}.$$

The characteristic canonical momentum scales as

$$p_{\text{atom}} \sim \frac{\hbar}{a_0} \propto X^{-1}.$$

Hamilton's equation gives the true-frame orbital velocity

$$v_T \sim \frac{X^2 p_{\text{atom}}}{m_e} \propto X.$$

Thus $v_T/(cX)$ is constant, and the velocity reported using co-scaling rods and fixed clocks is also constant.

These results establish a hydrogenic, adiabatic proxy only. They do not derive the scaling of QED corrections, molecular structure, hyperfine standards, nuclear scales, material rods, or strong/weak quantities.

Newtonian bound-orbit realization

Premises and Hamiltonian

Consider a test mass m in a quasi-static Newtonian potential generated by a fixed source content M_s . Use

$$G_{\text{eff}}(X) = G_N X$$

and the nonrelativistic Hamiltonian

$$H_{\text{orb}} = \frac{X^2 p^2}{2m} - \frac{G_{\text{eff}}(X) M_s m}{r}.$$

The orbit is assumed adiabatic, so its angular action

$$J = pr$$

is conserved.

Orbit scale

For a circular orbit, substitute $p = J/r$:

$$H_{\text{orb}}(r) = \frac{X^2 J^2}{2mr^2} - \frac{G_{\text{eff}} M_s m}{r}.$$

The stationary-radius condition is

$$\frac{dH_{\text{orb}}}{dr} = -\frac{X^2 J^2}{mr^3} + \frac{G_{\text{eff}} M_s m}{r^2} = 0.$$

Therefore

$$r = \frac{X^2 J^2}{m^2 G_{\text{eff}} M_s}.$$

For fixed J, m, M_s ,

$$r \propto \frac{X^2}{G_{\text{eff}}}.$$

Demanding the clean rod scaling $r \propto X$ fixes

$$\boxed{G_{\text{eff}}(X) = G_N X}.$$

Conversely, that coupling law makes the bound orbital radius co-scale with local rods:

$$\boxed{r(X) = X r_o}.$$

Orbital velocity and timing

Hamilton's equation gives

$$v_T = \frac{\partial H_{\text{orb}}}{\partial p} = \frac{X^2 p}{m}.$$

With $p = J/r \propto X^{-1}$,

$$\boxed{v_T \propto X}.$$

The true-frame orbital period scales as

$$T_T \sim \frac{r}{v_T} \propto X^0.$$

Fixed local clocks therefore count a constant number of ticks per orbit. The turning combination is

$$v_T^2 r = X^2 G_{\text{eff}} M_s \propto X^3,$$

where two powers come from the dressed kinetic relation and one from $G_{\text{eff}} \propto X$.

This is a Newtonian, adiabatic orbit realization. It does not derive a relativistic gravitational field equation, strong-field dynamics, lensing, or the cosmological gravitational source weight.

Conditional linear-growth correspondence

The same kinetic and coupling powers can be checked dynamically in the linear, quasi-static matter regime.

Premises

1. The homogeneous yield history $X(t)$ is prescribed, with $a_B = 1/X$ and $H = -\dot{X}/X$.
2. The true-frame chart is static for the homogeneous matter background, so $\bar{\rho}_T$ is constant.
3. Bound-frame volume conversion gives

$$\bar{\rho}_B = X^3 \bar{\rho}_T.$$

4. The density contrast δ is invariant under the homogeneous conversion.
5. Matter is nonrelativistic and pressureless; perturbations are linear.
6. Gravity is quasi-static and obeys

$$\Delta_T U_T = 4\pi G_N X \bar{\rho}_T \delta.$$

7. The content-velocity variable $\mathbf{u} \equiv \mathbf{p}/m$ obeys

$$\dot{\mathbf{x}}_T = X^2 \mathbf{u}, \quad \dot{\mathbf{u}} = -\nabla_T U_T.$$

Derivation

Linear continuity on the static true-frame grid is

$$\dot{\delta} = -X^2 \nabla_{T^\cdot} \mathbf{u}.$$

Differentiate:

$$\ddot{\delta} = -2X\dot{X}\nabla_{T^\cdot} \mathbf{u} - X^2 \nabla_{T^\cdot} \dot{\mathbf{u}}.$$

The continuity equation gives

$$-2X\dot{X}\nabla_{T^\cdot} \mathbf{u} = \frac{2\dot{X}}{X} \dot{\delta} = -2H\dot{\delta}.$$

Euler and Poisson give

$$-X^2 \nabla_{T^\cdot} \dot{\mathbf{u}} = X^2 \Delta_T U_T = 4\pi G_N X^3 \bar{\rho}_T \delta.$$

Since $X^3 \bar{\rho}_T = \bar{\rho}_B$,

$$\ddot{\delta} + 2H\dot{\delta} = 4\pi G_N \bar{\rho}_B \delta.$$

This is the standard linear matter-growth equation at the same prescribed $H(t)$ and $\bar{\rho}_B(t)$. The friction coefficient comes from differentiating the X^2 kinetic dressing. The source contains the same two kinetic powers plus the one coupling power in $G_{\text{eff}} = G_N X$.

The equality is a conditional correspondence, not a background prediction. It does not determine $H(z)$ and does not establish relativistic perturbations, radiation-era transfer, nonlinear growth, or lensing.

Realization ledger

Sector or check	Result in this appendix	Regime	Status beyond the regime
Local rods, clocks, and ranging	$x_B = x_T/X$, $\ell_T \propto X$, $\nu_T \propto X^0$, local light/radar ratios fixed	clean homogeneous branch	transform and target only
Electromagnetism	$e^2 = e_o^2 X$, $a_0 \propto X$, $\Delta E \propto X^0$, α_X fixed	hydrogenic, adiabatic proxy	QED, molecular, hyperfine, and material closure open
Gravity-bound systems	$G_{\text{eff}} = G_N X$, $r \propto X$, $v_T \propto X$	Newtonian, adiabatic, fixed source content	relativistic and strong-field closure open
Matter growth	standard linear equation at fixed $H(z)$	pressureless, linear, quasi-static	radiation, horizon-scale, nonlinear, and lensing sectors open
Strong sector	no realization derived	—	confinement and nuclear scaling open
Weak sector	no realization derived	—	transition-rate scaling open

The broader statement that all space-mediated couplings scale as X is therefore a hypothesis supported here only by the EM proxy and Newtonian gravity realization. It is not an established all-sector principle.

Local-drift falsifiers and scope

The clean realizations predict no drift in the dimensionless quantities they actually close:

- α_X and hydrogenic electronic spectral ratios remain fixed;
- local light/radar ranging ratios remain fixed;
- Newtonian orbital periods remain fixed relative to the matched clock;
- co-scaled atomic and orbital lengths remain fixed relative to the same rods.

A confirmed nonzero drift that survives reduction to a dimensionless local comparison would falsify the corresponding clean realization. Relevant channels include cross-family clock ratios, $\Delta\alpha/\alpha$, recoil-to-spectral ratios, radar-to-atomic length ratios, orbital-to-atomic timing, and composition-dependent residuals.

Because strong and weak realization is not derived here, an anomaly involving nuclear, confinement, or weak-transition physics would expose an open closure problem before it could be called a falsification of the χ^2 generator itself. Likewise, failure of $G_{\text{eff}} \propto X$ in the Newtonian mapping would reject that scoped gravity realization without undoing the free-carrier theorem of Derivation 5.2A.

Result and boundaries

This appendix establishes a mutually consistent clean-branch package under explicit premises:

$$\boxed{x_B = \frac{x_T}{X}, \quad a_B = \frac{1}{X}, \quad \ell_T \propto X, \quad \nu_T \propto X^0},$$

$$\boxed{e^2 \propto X, \quad a_0 \propto X, \quad \alpha_X = \text{constant}} \quad (\text{hydrogenic proxy}),$$

and

$$\boxed{G_{\text{eff}} \propto X, \quad r_{\text{orbit}} \propto X} \quad (\text{Newtonian orbit}).$$

With the additional linear and quasi-static premises, it also gives

$$\boxed{\ddot{\delta} + 2H\dot{\delta} = 4\pi G_N \bar{\rho}_B \delta}$$

at a fixed background history.

It does not establish exact local indistinguishability, all-sector coupling co-scaling, Friedmann dynamics, a dressed gravitational source law, strong/weak closure, radiation physics, or a

complete observational cosmology. Chapter 5.6 may cite the scoped EM, gravity, and linear-growth results only with these regime boundaries attached.

Manuscript Status

This book is developed in the open and in order, so not every chapter carries the same weight. The table below records, per chapter, how mature the material is and what level of verification it has passed. The tiers themselves are defined in the project’s verification policy; here they are only reported.

Two distinctions matter when reading it. *Maturity* describes how settled the writing and structure are, from an early **seed** through a **working draft** and a **stabilizing** chapter to **mature** material. *Verification* describes whether a chapter has passed a distinct review pass — editorial (**V1**), conceptual (**V2**), or technical (**V3**) — and a dash means no formal pass is yet recorded. Contribution and verification are separate: that a chapter exists says nothing about whether it has been checked. Read chapters that are early in maturity or unverified as provisional.

Chapter	Maturity	Verification	Notes
Introduction	Stabilizing	V2 (conceptual) · AI-assisted review	Framing and motivation; conceptually reviewed, but not load-bearing for later derivations.
Foundations	Stabilizing	V3 (technical) · AI-assisted review	Primitive variables, ADMC, and the invertible mappings — the load-bearing locks.

Chapter	Maturity	Verification	Notes
Gravity and Structured Spacetime	Stabilizing	V2 (conceptual) · AI-assisted review	Source-to-field-to- observer pipeline; historical V2 review with targeted V3 chains. Five narrow companion files received independent technical review with the 2026-10-01 appendix reconciliation; this is not whole-chapter re-verification.
Quantum Mechanics in Momentum Language	Working draft	V2 (conceptual) · AI-assisted review	Momentum-language reconstruction; correspondence-heavy, read provisionally.
Space Expansion and Stage Dressing	Working draft	V2 (conceptual) · AI-assisted review	Basic-slot stage dressing, ² massive motion, and scoped cosmological correspondence at a supplied history.
A First Picture of M1 Geometry	Working draft	—	Adopted opening chapter of Part 0 (2026-10-01, v17). Earlier conceptual, writing and local mathematical reviews cover named revisions and constructions; no whole-v17 independent verification is claimed.

Chapter	Maturity	Verification	Notes
Derivation appendices	Working draft	V2 (conceptual) · AI-assisted review	Historical aggregate V2 status is retained, not re-certified. Gravity appendices 3.3A–3.10A received exact-revision independent V3 review on 2026-10-01; QM and other appendices were not re-accepted by this review.

The data above is generated from a single canonical ledger (`manuscript/metadata/manuscript-status.yml`) in the project repository, which also holds the per-chapter contribution notes. Authorship and contribution policy is described in the Preface; the verification tiers are defined in the project’s verification policy.

7 References

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